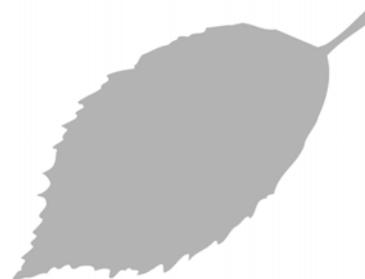


**Massachusetts  
Tests for Educator Licensure<sup>®</sup> (MTEL<sup>®</sup>)**



**Elementary  
Mathematics (53)**

**PRACTICE TEST**



**[www.mtel.nesinc.com](http://www.mtel.nesinc.com)**

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## INTRODUCTION

This document is a printable version of the Massachusetts Tests for Educator Licensure® (MTEL®) Elementary Mathematics (53) Online Practice Test. This practice test is a sample test consisting of 100 multiple-choice questions and 2 open-response item assignments.

To assist you in recording and evaluating your responses on the practice test, a Multiple-Choice Answer Sheet, an Answer Key Worksheet, and an Evaluation Chart by test objective are included for the multiple-choice questions. A blank Response Sheet, Evaluation Information, and Sample Responses and Analyses, as well as a Scoring Rubric, are included for the open-response items. Lastly, there is a Practice Test Score Calculation worksheet.

## PURPOSE OF THE PRACTICE TEST

The practice test is designed to provide an additional resource to help you effectively prepare for the MTEL Elementary Mathematics (53) test. The primary purpose of the practice test is to help you become familiar with the structure and content of the test. It is also intended to help you identify areas in which to focus your studies. Education faculty and administrators of teacher preparation programs may also find this practice test useful as they help students prepare for the official test.

## TAKING THE PRACTICE TEST

In order to maximize the benefits of the practice test, it is recommended that you take this test under conditions similar to the conditions under which the official MTEL tests are administered. Try to take the practice test in a quiet atmosphere with few interruptions and limit yourself to the four-hour time period\* allotted for the official test administration. You will find your results to be more useful if you refer to the answer key only after you have completed the practice test. Note that use of a calculator for the MTEL Elementary Mathematics (53) test is prohibited at the test administration.

## INCORPORATING THE PRACTICE TEST IN YOUR STUDY PLAN

Although the primary means of preparing for the MTEL is your college education, adequate preparation prior to taking or retaking the MTEL test is strongly recommended. How much preparation and study you need depends on how comfortable and knowledgeable you are with the content of the test.

The first step in preparing to take the MTEL is to identify what information the test will address by reviewing the objectives for your field. A complete, up-to-date list of the Test Objectives is included in the Test Information Booklet for each test field. The test objectives are the core of the testing program and a helpful study tool. Before taking or retaking the official test, focus your study time on those objectives for which you wish to strengthen your knowledge.

This practice test may be used as one indicator of potential strengths and weaknesses in your knowledge of the content on the official test. However, because of potential differences in format and difficulty between the practice test and an official MTEL Elementary Mathematics (53) test, it is not possible to predict precisely how you might score on an official MTEL Elementary Mathematics (53) test. Keep in mind that the subareas for which the test weighting is greatest will receive emphasis on this test. Refer to the Test Information Booklet for additional information about how to prepare for the test.

\* For the Communication and Literacy Skills and General Curriculum tests, candidates may take one or both subtests during the four-hour session.

**ELEMENTARY MATHEMATICS  
PRACTICE TEST**

## GENERAL TEST DIRECTIONS

This practice test consists of two sections: (1) a multiple-choice question section and (2) an open-response item assignment section. Each multiple-choice question on the practice test has four answer choices. Read each question carefully and choose the ONE best answer. Record each answer on the answer sheet provided.

- Sample Question:
1. What is the capital of Massachusetts?
    - A. Worcester
    - B. New Bedford
    - C. Boston
    - D. Springfield

The correct answer to this question is C. You would indicate that on the answer sheet.

The open-response section of this practice test requires written responses. Directions for the open-response item assignments appear immediately before those assignments.

You may work on the multiple-choice questions and open-response item assignments in any order that you choose. You may wish to monitor how long it takes you to complete the practice test. When taking the actual MTEL Elementary Mathematics (53) test, you will have one four-hour test session in which to complete the test.

**MULTIPLE-CHOICE ANSWER SHEET**

<b>Question Number</b>	<b>Your Response</b>
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2	
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<b>Question Number</b>	<b>Your Response</b>
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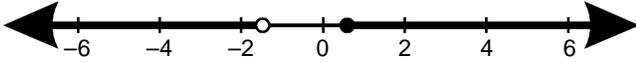
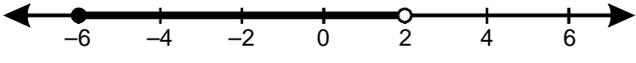
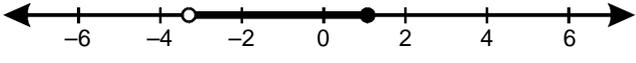
<b>Question Number</b>	<b>Your Response</b>
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**MULTIPLE-CHOICE QUESTIONS**

1. Given that  $a$ ,  $b$ , and  $c$  are integers and that  $a \leq b$  and  $b > c$ , which of the following number lists could represent  $a$ ,  $b$ , and  $c$  respectively?

- A. 1, 1, 2
- B. -3, -2, 1
- C. -1, -2, 3
- D. -2, -1, -2

2. Which of the following number lines represents the solution set for the inequality  $-3 \leq 2x < 1$ ?

- A. 
- B. 
- C. 
- D. 

3. Which of the following expressions is equivalent to  $a^{-5} \cdot b^{\frac{2}{3}}$ ?

A.  $\frac{\sqrt[3]{b^2}}{a^5}$

B.  $(-a^5)(2 \cdot \sqrt[3]{b})$

C.  $-10\frac{\sqrt{a}}{b^3}$

D.  $(-5\sqrt{a})(\sqrt[3]{b})^2$

4. A photographer sells postcards at a craft fair. The photographer calculates the cost of 12 postcards. The price of each of the postcards is the same and there is no sales tax. The photographer calculates the total cost of the postcards as \$29.58, but recognizes that this total is incorrect because if a number is divisible by 12, then:

A. the sum of the digits must be divisible by both 3 and 4 and the last digit must be a factor of 6.

B. the last two digits must be divisible by 12.

C. the sum of the digits must be divisible by 3 and the last two digits must be divisible by 4.

D. the last two digits must be divisible by 6.

5. Use the diagram below to answer the question that follows.



 = 2

 = 1

 = 0

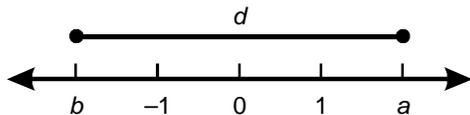
A teacher is using cards to model numbers in base three. What base-ten number is represented by the diagram?

- A. 91
  - B. 93
  - C. 271
  - D. 273
6. When two different prime numbers are multiplied, it is possible for the product to be:
- A. an even number.
  - B. a perfect square.
  - C. an irrational number.
  - D. a negative integer.

7. Which of the following equations best illustrates the concept of the additive inverse?
- A.  $4 + 0 = 4 - 0$
  - B.  $3 + (-3) = 0$
  - C.  $4 + 2 = (4 - 2) + 4$
  - D.  $3 - (-1) = 3 + 1$
8. What is the value of the expression  $6 + 2 \times (3 - 1)^2 + 2 \div 4$ ?
- A. 4
  - B. 6
  - C.  $10\frac{1}{2}$
  - D.  $14\frac{1}{2}$
9. A community group is selling tickets to a spaghetti supper fundraising event. Adult tickets cost \$9 each and children's tickets cost \$6. The cost of food and supplies for the event is \$197. If the goal is to make a \$1600 profit from the spaghetti supper, and if 120 adult tickets are sold, how many children's tickets will need to be sold in order to reach the goal?
- A. 36
  - B. 54
  - C. 87
  - D. 120

10. What is the value of the expression  $\frac{7 - (3 - 14) \times (-2)}{(8 - 12) - (2 - 5)}$  ?
- A. -29
  - B. -15
  - C. 15
  - D. 29

11. Use the number line below to answer the question that follows.



- Which of the following expressions represents the length of line segment  $d$ ?
- A.  $|a + b|$
  - B.  $|a| + |b|$
  - C.  $a + b$
  - D.  $|a| - |b|$
12. Multiplying by 3, adding 2, and dividing by 5 is the inverse of which of the following operations?
- A. dividing by 3, subtracting 2, and multiplying by 5
  - B. multiplying by  $\frac{3}{5}$  and subtracting 2
  - C. multiplying by 5, subtracting 2, and dividing by 3
  - D. subtracting 2 and dividing by  $\frac{5}{3}$

13. If a fraction  $\frac{a}{b}$  is simplified to its lowest terms, which of the following must be true?
- A.  $a$  or  $b$  is a prime number.
  - B.  $a$  is less than  $b$ .
  - C.  $a$  equals 1 and  $b$  does not equal 0.
  - D.  $a$  and  $b$  have only 1 as a common factor.

14. **Use the number sentence below to answer the question that follows.**

$$11 \div 4 = 2 \text{ remainder } 3$$

The remainder shown above can be represented by which of the following?

- A.  $\frac{3}{2}$
- B.  $\frac{3}{4}$
- C.  $\frac{2}{3}$
- D.  $\frac{4}{11}$

15. Use the expression below to answer the question that follows.

$$2\frac{1}{2} \times \frac{1}{4}$$

Which of the following problems can be solved using the operation above?

- A. A person has  $2\frac{1}{2}$  gallons of paint and uses  $\frac{1}{4}$  of it to paint a wall. How much paint was used?
- B. A bicyclist travels  $2\frac{1}{2}$  miles in  $\frac{1}{4}$  of an hour. How many miles per hour is the bicyclist traveling?
- C. A rectangle has a length of  $2\frac{1}{2}$  inches. If the width of the rectangle is  $\frac{1}{4}$  of the length, what is the area of the rectangle?
- D. A bag contains  $2\frac{1}{2}$  cups of chocolate chips. If one batch of a recipe calls for  $\frac{1}{4}$  cup of chocolate chips, how many batches can be made?
16. Maria ran  $7\frac{1}{2}$  miles on Monday. On Wednesday she ran  $\frac{4}{5}$  the distance that she ran on Monday and on Saturday she ran  $1\frac{1}{2}$  times the distance she ran on Wednesday. If Monday, Wednesday, and Saturday were the only days Maria ran, what was the total distance that she ran during the week?
- A.  $21\frac{7}{8}$  miles
- B.  $22\frac{1}{2}$  miles
- C.  $23\frac{1}{8}$  miles
- D.  $24\frac{3}{4}$  miles

17. Which of the following equations represents a valid algorithm?

A.  $2\frac{3}{4} + 3\frac{1}{3} = (2 + 3) + \left(\frac{3}{4} + \frac{1}{3}\right)$

B.  $2\frac{3}{4} \times 3\frac{1}{3} = (2 \times 3) + \left(\frac{3}{4} \times \frac{1}{3}\right)$

C.  $2\frac{3}{4} \div 3\frac{1}{3} = (2 \div 3) + \left(\frac{3}{4} \div \frac{1}{3}\right)$

D.  $2\frac{3}{4} - 3\frac{1}{3} = (2 - 3) - \left(\frac{3}{4} - \frac{1}{3}\right)$

18. Which of the following expressions is equivalent to  $1 + \frac{1}{\frac{a}{2}}$  for  $a \neq 0$ ?

A.  $2a + 1$

B.  $2a + 2$

C.  $\frac{3}{2a}$

D.  $\frac{2}{3a}$

19. In a particular adult skeleton, the humerus is 36.46 centimeters in length. The hand measures 18.9 centimeters. How much longer is the humerus than the hand?

A. 17.56 cm

B. 22.56 cm

C. 34.57 cm

D. 35.43 cm

20. The skeleton represents approximately 35% of the human body's total weight. A particular body weighs 130 pounds, 23 pounds of which are contributed by water contained in the skeleton. Approximately what percentage of this skeleton is composed of water?
- A. 8%
  - B. 18%
  - C. 46%
  - D. 50%

21. Use the figures below to answer the question that follows.

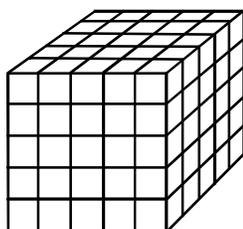


Figure 1

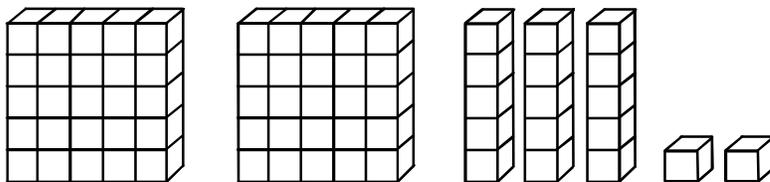


Figure 2

The large cube in Figure 1 represents one unit. What is the decimal representation of Figure 2?

- A. 0.232
- B. 0.248
- C. 0.536
- D. 0.680

22. A tool company produces a bolt for which the specifications require a diameter that measures between 0.997 centimeters and 1.03 centimeters. Which of the following pairs of measurements are for bolts that are within the specifications?
- A. 0.979 cm and 0.999 cm
  - B. 0.998 cm and 1.033 cm
  - C. 1.005 cm and 0.999 cm
  - D. 1.20 cm and 1.002 cm
23. Which of the following numbers represents the exact value of  $(0.\overline{4})(18)$ ?
- A. 7.2
  - B.  $\frac{198}{25}$
  - C. 7.992
  - D. 8
24. A retailer puts out-of-season clothing on sale for 40% off the regular price,  $r$ . Each subsequent week, the retailer reduces the sale price by an additional 10%. Which of the following expressions represents the sale price during the third week of the sale?
- A.  $0.324r$
  - B.  $0.486r$
  - C.  $0.514r$
  - D.  $0.996r$

25. A scatter plot would be most useful in presenting which of the following types of data?
- A. data tracking school attendance and performance on standardized tests
  - B. data listing the age at which children in an elementary school lost their first primary tooth
  - C. data recording the height of a student from kindergarten through fifth grade
  - D. data comparing math test scores for two sets of third-grade students
26. Use the information below to answer the question that follows.

67.0
5.5
-12.7
-124.4

The numbers in the box above represent the melting points, in degrees Celsius, for four chemical compounds. Which of the following statistics will be most affected by including a compound with a melting point of  $13.0^{\circ}\text{C}$  in the list?

- A. median
- B. mode
- C. mean
- D. range

27. Use the information below to answer the question that follows.

902, 981, 1004, 1007, 1015, 1075, 1108, 1113, 1153, 1158,  
1163, 1171, 1174, 1199, 1248, 1256, 1282, 1294, 1408, 1478

The weights in pounds (lb.) of twenty giant pumpkins that were entered into a contest are shown in the box above. Which of the following tables represents a properly constructed frequency distribution that organizes the data into five classes?

A.

Weight (lb.)	Frequency
875–1000	2
1001–1126	6
1127–1252	7
1253–1378	3
1379–1500	2

B.

Weight (lb.)	Frequency
825–999	2
1000–1099	4
1100–1299	12
1300–1449	1
1450–1525	1

C.

Weight (lb.)	Frequency
901–1020	5
1021–1140	3
1141–1260	8
1261–1380	2
1381–1500	2

D.

Weight (lb.)	Frequency
801–1000	2
1001–1100	4
1101–1200	8
1201–1300	4
1301–1500	2

28. Use the information below to answer the question that follows.

127, 129, 130, 134, 139, 143, 145, 148, 149, 151, 153

The numbers in the box above represent the heights in centimeters of ten-year-old boys. Based on the data provided, the boy with a height of 145 centimeters is in approximately which of the following percentiles?

- A. 36th
- B. 55th
- C. 64th
- D. 70th

29. Use the information below to answer the question that follows.

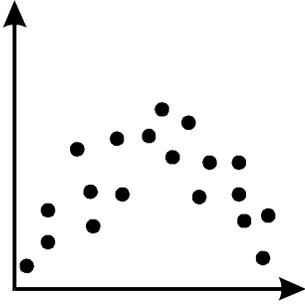
How much time per day do you spend using a computer or cell phone for recreational purposes (e.g., e-mailing, social networking, shopping, surfing the Web)?

This survey question was sent to 4000 randomly selected e-mail addresses across the United States. Based on the results of 2500 responses, the authors of the survey concluded that people in the United States spend an average of 3.5 hours per day using a computer or cell phone for recreational purposes. Which of the following flaws in the survey undermines the validity of the authors' conclusion?

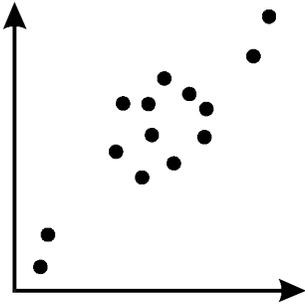
- A. The survey question provides specific examples of recreational computer use.
- B. The responses of sample populations should not be used to draw conclusions about entire populations.
- C. The number of responses represents too small a return rate to generate statistically meaningful data.
- D. The sample population is not representative of the entire population of the United States.

30. Which of the following scatter plots would generate the line of best fit that would be most useful in making accurate predictions about the data?

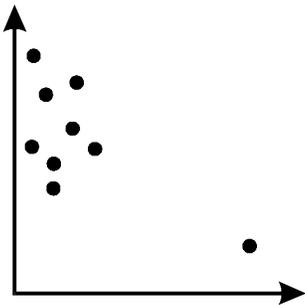
A.



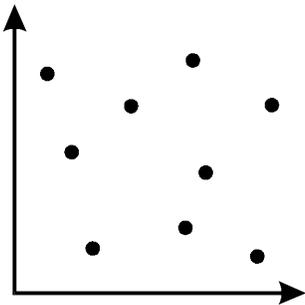
B.



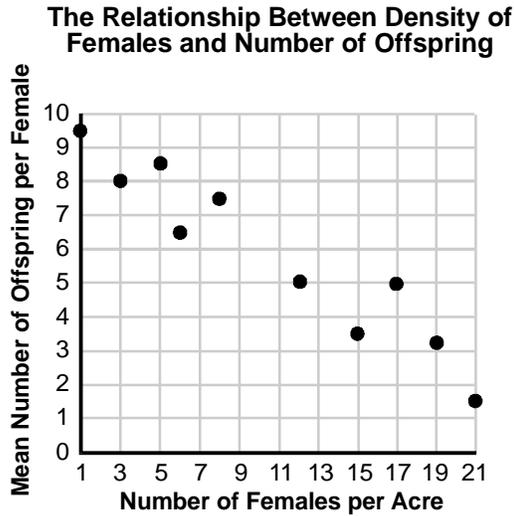
C.



D.



31. Use the graph below to answer the question that follows.



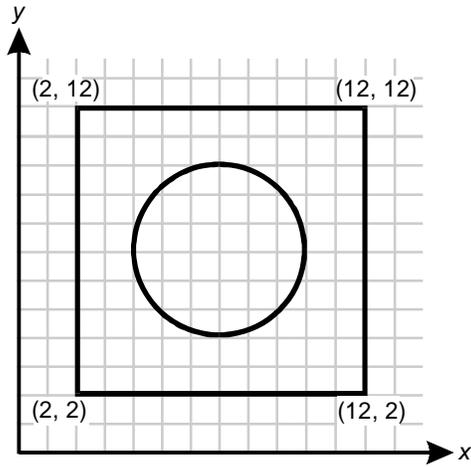
A biologist is studying the factors that limit the population size of a small mammal in a woodland. Which of the following statements is best supported by the data in the graph?

- A. Five females per acre is the density that will sustain the current population size.
  - B. Factors other than density had a greater effect on the number of offspring produced.
  - C. Approximately six offspring would be born to females in areas with a density of nine females per acre.
  - D. Twenty-two females per acre would be too great to support the production of any offspring.
32. In how many different ways can the letters in the word Boston be arranged?
- A. 12
  - B. 15
  - C. 360
  - D. 720

33. Five employees have volunteered to work the first shift at a bakery for the coming week. The manager only needs two employees to work at that time. If the manager selects the employees to work the first shift randomly, what is the probability that two particular employees will work together?
- A. 5%
  - B. 10%
  - C. 20%
  - D. 40%
34. A bowl of fruit consists of five apples, seven oranges, four pears, and two plums. If a person randomly selects and eats one fruit per day for three days, what is the probability that an apple will be the fruit selected on all three days? (Assume that the selected fruit is not replaced during the three-day period.)
- A. 0.0123
  - B. 0.0214
  - C. 0.170
  - D. 0.278

35. A full deck of playing cards consists of 52 cards divided equally into four suits: hearts, diamonds, spades, and clubs. Which of the following simulations would generate data to estimate the probability of a player receiving four cards of the same suit when the cards are dealt from a full deck?
- A. Represent each suit with a number from 1 to 4 and use a computer to randomly generate 52 four-digit numbers using only the numbers 1–4.
  - B. Label all the sides of four six-sided dice as either hearts, diamonds, spades, or clubs, and simultaneously roll the dice 100 times.
  - C. Designate each sector of a four-sector spinner as a heart, diamond, spade, or club, and spin the arrow 52 times.
  - D. Randomly select four marbles from a bag containing 13 marbles of each of four different colors. Repeat 100 times.

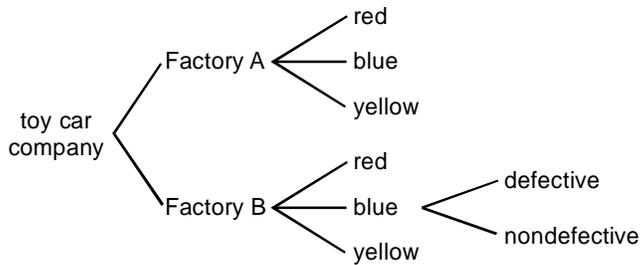
36. Use the diagram below to answer the question that follows.



If a point is randomly selected within the square shown in the diagram above, what is the approximate probability that it will be a point within the circle?

- A. 0.283
- B. 0.395
- C. 0.470
- D. 0.708

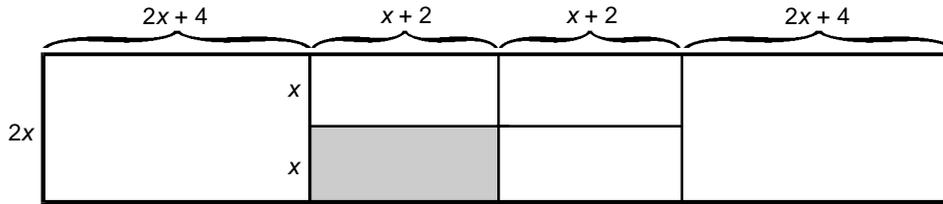
37. Use the diagram below to answer the question that follows.



The diagram above represents the production of toy cars by a company with two factories. Sixty percent of the cars are manufactured in Factory A and forty percent of the cars are manufactured in Factory B. Both factories manufacture three types of cars—a red model, a blue model, and a yellow model. In Factory A, 35% of the cars manufactured are red, 20% are blue, and 45% are yellow. In Factory B, 45% of the cars are red, 35% are blue, and 20% are yellow. A quality control manager has learned that 15% of the blue cars manufactured in Factory B during the previous month are defective. What is the probability that a car selected at random from all the cars manufactured during the previous month is defective?

- A. 0.021
- B. 0.053
- C. 0.141
- D. 0.150

38. Use the diagram below to answer the question that follows.



The diagram above represents a garden that is divided into rectangular plots with dimensions as indicated. A butterfly lands at random in the garden. What is the probability that the butterfly lands within the shaded part of the garden?

- A.  $\frac{1}{6}$
- B.  $\frac{1}{8}$
- C.  $\frac{1}{10}$
- D.  $\frac{1}{12}$

39. Use the figures below to answer the question that follows.



Figure 1

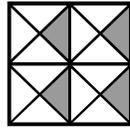


Figure 2

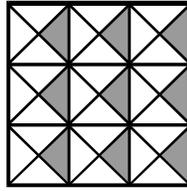


Figure 3

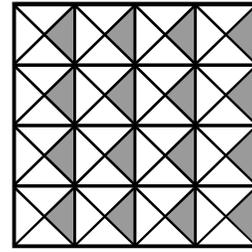
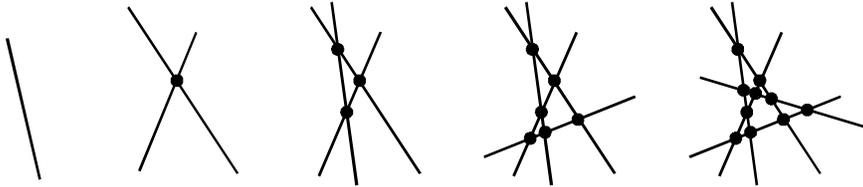


Figure 4

The four squares in the series above contain shaded and unshaded triangles. If this pattern continues, how many unshaded triangles will there be in the seventh figure in the series?

- A. 66
- B. 81
- C. 108
- D. 147

40. Use the figures below to answer the question that follows.



In the sequence of figures above, nonparallel lines have been drawn so that no three lines pass through the same point. Which of the following equations represents the relationship between the number of nonparallel lines,  $n$ , and the number of intersection points,  $p$ , in each of the five figures?

- A.  $p = \frac{n(n-1)}{2}$
- B.  $p = \frac{(n^2-1)}{n+1}$
- C.  $p = \frac{2(n^2-1)}{3n}$
- D.  $p = \frac{2(n-1)}{n+1}$

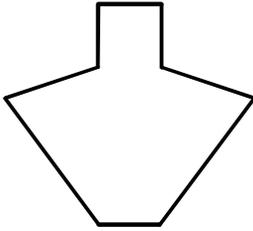
41. Use the table below to answer the question that follows.

Distance from Light Source, $x$	1	2	3	4	5
Intensity of Light, $f(x)$	1	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{16}$	$\frac{1}{25}$

The table above shows how the intensity of light changes with increased distance from the point of the light source. Which of the following functions best models the relationship between the distance from the light source and the intensity of the light?

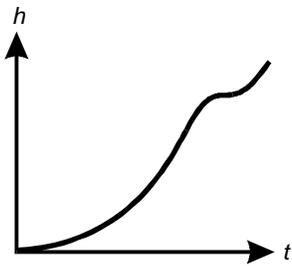
- A.  $f(x) = x^2 + bx + c$
- B.  $f(x) = mx + b$
- C.  $f(x) = x^2$
- D.  $f(x) = \frac{1}{x^2}$

42. Use the diagram below to answer the question that follows.

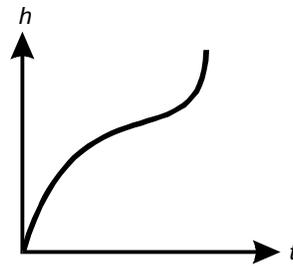


Water is poured at a constant rate into a container with the shape shown above. Which of the following graphs best represents the height,  $h$ , of the water as a function of time,  $t$ ?

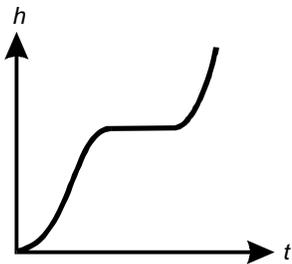
A.



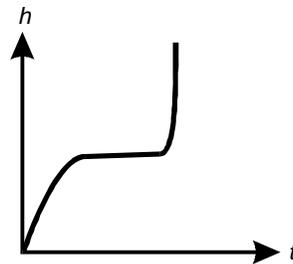
B.



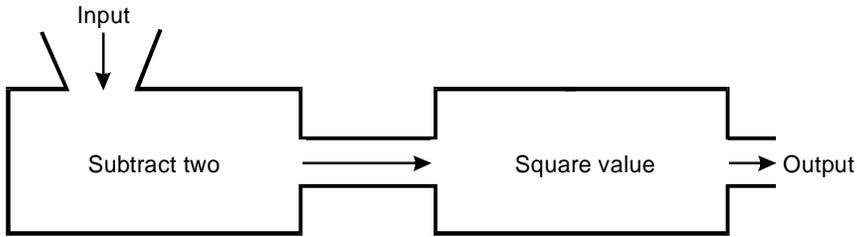
C.



D.

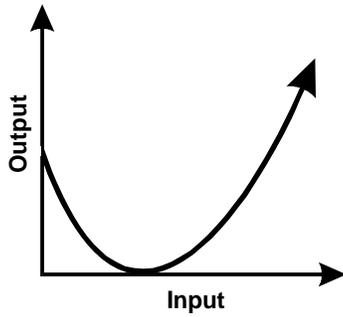


43. Use the diagram below to answer the question that follows.

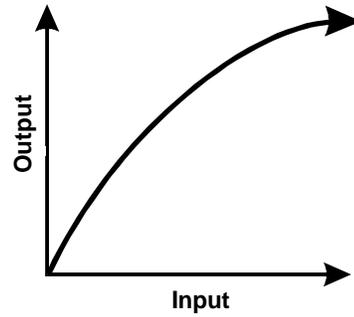


Which of the following graphs models the input and output of the function machine shown in the diagram above?

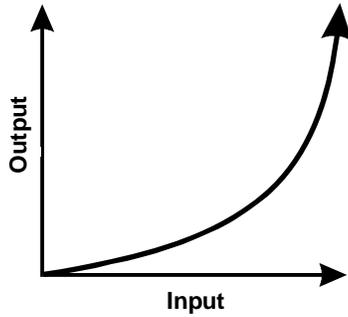
A.



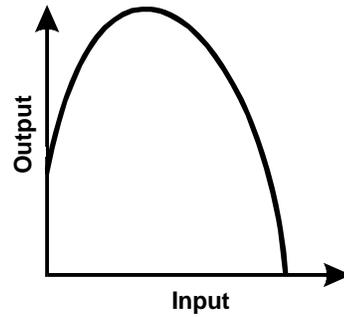
B.



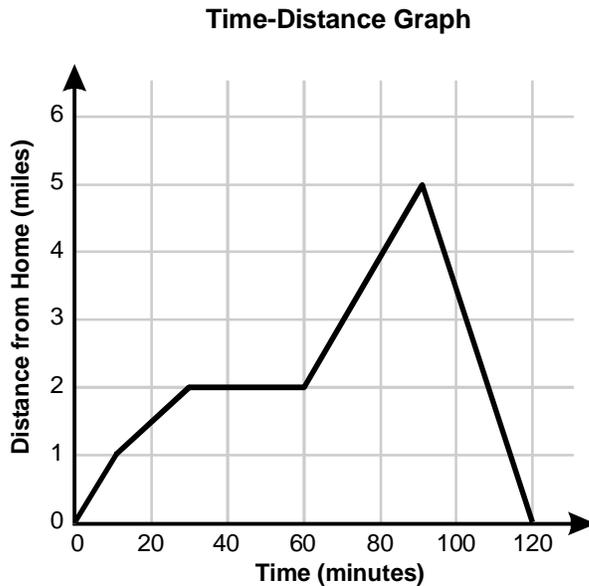
C.



D.



44. Use the graph below to answer the question that follows.



The graph above best represents which of the following scenarios?

- A. A bicyclist leaves home and travels 1 mile during the first 10 minutes and then 2 miles in the next 10 minutes. The cyclist then stops for a 30-minute break before continuing for another 30 minutes on a 3-mile rough dirt road. Finally, the cyclist rides for 30 minutes on a 5-mile direct route home.
- B. A bicyclist leaves home and travels 1 mile during the first 10 minutes and then an additional mile uphill in the next 20 minutes. The cyclist then bicycles out a short side road and returns to the main road before cycling another 30 minutes on a 2-mile rough dirt road and returning home along a direct 5-mile route.
- C. A bicyclist leaves home and travels 1 mile on a paved road during the first 10 minutes and an additional mile on a rough dirt road during the next 20 minutes. The cyclist then stops for 30 minutes before continuing 3 miles down a paved road to a river. Finally, the cyclist returns home on a direct paved road, completing the trip in 2 hours.
- D. A bicyclist leaves home and travels 1 mile during the first 10 minutes and then an additional 2 miles in the next 20 minutes. The cyclist then stops for a 30-minute break before continuing, traveling on a 3-mile rough dirt road for 30 minutes. Finally, the cyclist rides home on a direct route on a paved road, completing the trip in 2 hours.

45. A person's job is to inspect hand-painted glassware before pieces are packed for shipping. At the start of the work shift, there are 18 glasses ready for inspection. The person can inspect 6 glasses every hour. If throughout the workday 4 additional glasses become ready for inspection each hour, which of the following equations represents the number of glasses,  $g$ , that are waiting for inspection after  $h$  hours of the person's shift at work?

A.  $g = 2(9 - 5h)$

B.  $g = \frac{9 - 5h}{2}$

C.  $g = 2(9 - h)$

D.  $g = \frac{9 - 3h}{2}$

46. The average speed,  $s$ , for a round trip is given by  $s = \frac{2}{\frac{1}{a} + \frac{1}{b}}$ ,

where  $a$  is the average speed going to the destination, and  $b$  is the average speed for the return trip. What is the average speed for a round trip when the average speed going to the destination is 40 miles per hour and the average speed for the return trip is 60 miles per hour?

A. 24 miles per hour

B. 48 miles per hour

C. 50 miles per hour

D. 56 miles per hour

47. Which of the following statements describes the relationships between the variables in the formula  $C = \frac{k\sqrt{t}}{m}$ ?
- A.  $C$  varies jointly with  $\sqrt{t}$  and  $m$ .
  - B.  $C$  varies jointly with  $k$ ,  $\sqrt{t}$ , and  $m$ .
  - C.  $C$  varies directly with  $\sqrt{t}$  and inversely with  $\frac{1}{m}$ .
  - D.  $C$  varies directly with  $\sqrt{t}$  and inversely with  $m$ .
48. Which of the following equations expresses  $a_n = a_1 + dn - d$  when it is solved for  $n$ ?
- A.  $n = a_n + a_1 - 2d$
  - B.  $n = \frac{a_n - d + a_1}{d}$
  - C.  $n = 1 + \frac{a_n - a_1}{d}$
  - D.  $n = \frac{a_1 - d}{a - d}$
49.  $x + 3$  is one factor of  $x^3 + x^2 - 14x - 24$ . Which of the following expressions is another factor?
- A.  $x - 8$
  - B.  $x + 4$
  - C.  $x - 3$
  - D.  $x + 2$

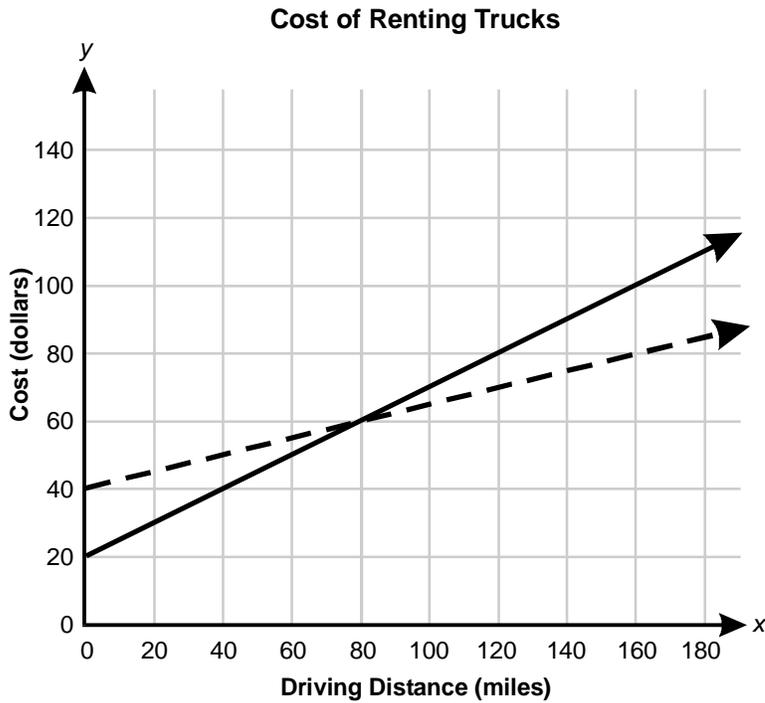
50. Use the problem below to answer the question that follows.

One-eighth of a number plus one-third of the number plus one-sixth of the number plus three equals the number. What is the number?

Which of the following equations could be an intermediate step in an algebraic solution to the problem presented in the box above?

- A.  $9x + 72 = 18x$
- B.  $12x + 48 = 18x$
- C.  $15x + 72 = 24x$
- D.  $18x + 48 = 24x$
51. What is the equation of a line that contains the point  $P = (-15, 8)$  and has a slope of  $\frac{3}{5}$ ?
- A.  $y = \frac{3}{5}x + 17$
- B.  $y = 3x + 23$
- C.  $y = \frac{3}{5}x - 7$
- D.  $y = 3x - 23$

52. Use the graph below to answer the question that follows.



Key	
	12 ft. truck
	20 ft. truck

In the graph above, analysis of which of the following characteristics of the lines would allow the comparison of the driving costs per mile for a 12-foot versus a 20-foot truck?

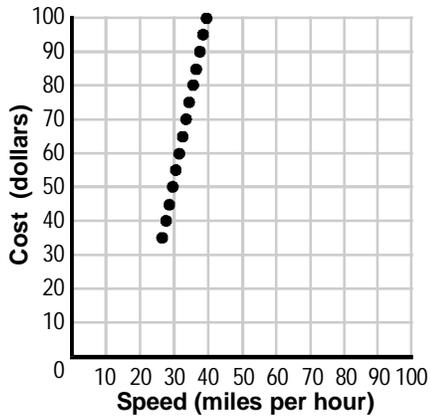
- A. the x-intercepts
- B. the slopes
- C. the y-intercepts
- D. the intersection

53. In which of the following equations does the dependent variable vary directly with the independent variable?

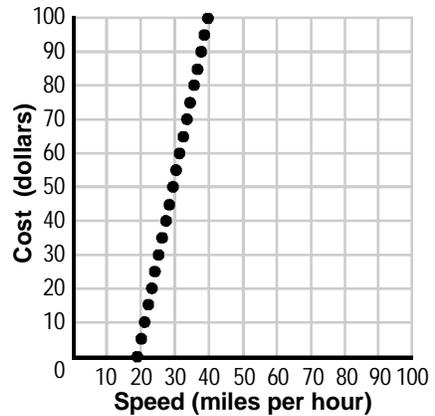
- A.  $yx = 2$
- B.  $y = 3x + 9$
- C.  $yx = x + 1$
- D.  $y = 7x$

54. The speed limit in the center of a town is 25 miles per hour. Drivers who are stopped for speeding are fined \$5 per mile in excess of the speed limit, in addition to a flat fee of \$30. Which of the following graphs represents the cost of fines for speeding in the town center?

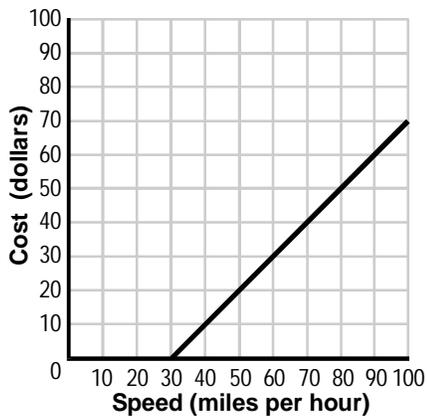
A.



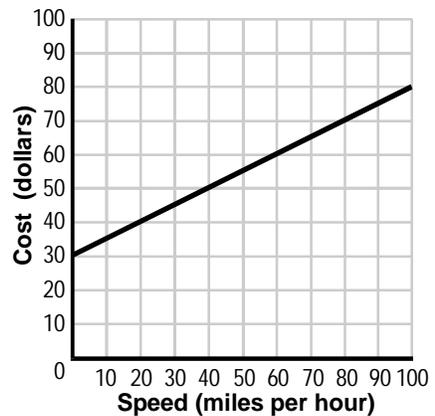
B.



C.



D.



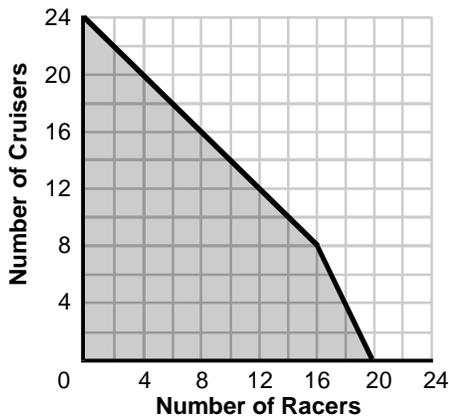
55. Two people live in towns that are 285 miles apart. They drive toward each other for  $t$  hours until they meet and stop for lunch. The person driving east averages  $r$  miles per hour, which is 15 miles per hour more than the person driving west. Which of the following expressions allows calculation of the speed of the eastbound driver?
- A.  $2rt + 15t = 285$
  - B.  $rt + 15t = 285$
  - C.  $2rt - 15t = 285$
  - D.  $rt - 15t = 285$

56. Use the problem below to answer the question that follows.

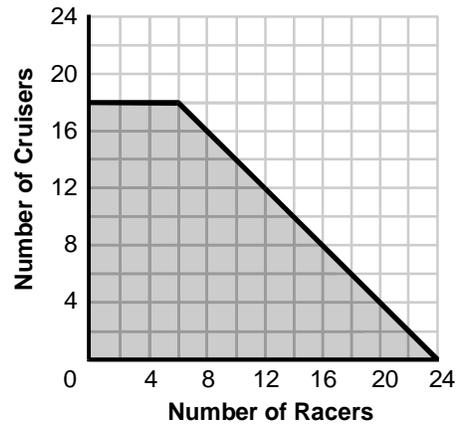
A company designs and sells two types of bicycles, the Racer and the Cruiser. The company can produce no more than 24 bicycles each month, using up to 160 hours total labor. It takes 8 hours to make a Racer and 4 hours to make a Cruiser. How many of each type of bicycle can the company make in a month?

The shaded area of which of the following graphs represents the solution to this problem?

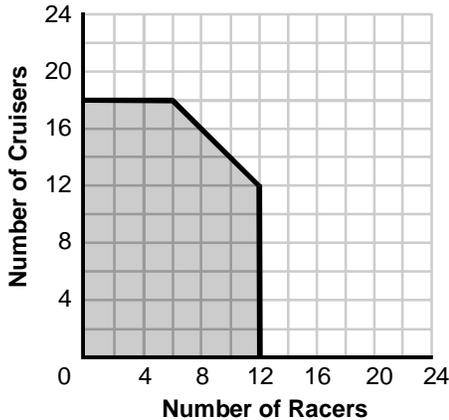
A.



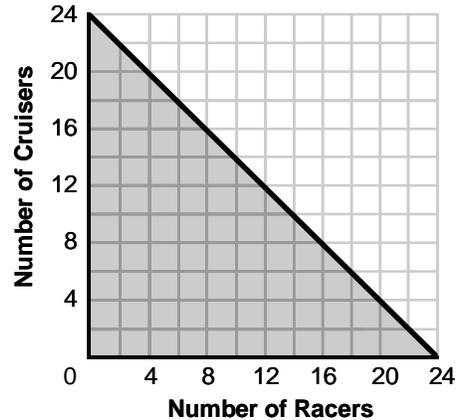
B.



C.



D.



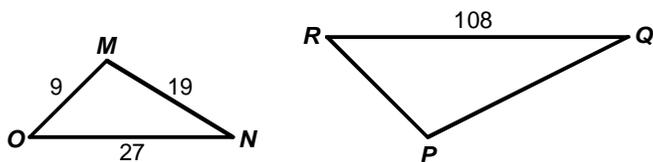
57. Use the equation below to answer the question that follows.

$$48x = 12y - 17 - 3x$$

What is the  $x$ -intercept of the graph of this equation?

- A.  $-3$
- B.  $-\frac{1}{3}$
- C.  $\frac{17}{45}$
- D.  $\frac{17}{12}$
58. If  $r$ ,  $s$ ,  $t$ , and  $u$  are variables, which of the following equations makes the relationship  $\frac{r}{s} = \frac{t}{u}$  a proportion?
- A.  $r + u = t + s$
- B.  $ur = st$
- C.  $r + t = u + s$
- D.  $tr = su$

59. Use the diagram below to answer the question that follows.



In the diagram above,  $\triangle OMN$  is similar to  $\triangle RPQ$ . Which of the following proportions calculates  $PQ$ ?

- A.  $\frac{108}{PQ} = \frac{27}{9}$
- B.  $\frac{PQ}{19} = \frac{108}{9}$
- C.  $\frac{27}{PQ} = \frac{19}{108}$
- D.  $\frac{PQ}{108} = \frac{19}{27}$

60. Use the problem below to answer the question that follows.

Steve lives in Boston and is planning a trip to Montreal. The driving distance between Boston and Montreal on the map is 9.3 inches. The scale of the map is 50 miles per inch and a half. If Steve's car averages 33 miles per gallon, and gas is \$2.60 per gallon, what will be his gas cost to go from Boston to Montreal?

Which of the following equations should be used to determine the cost of gas,  $g$ , for the trip to Montreal?

- A.  $g = \frac{(9.3)(50)}{(1.5)} \times \frac{1}{33} \times 2.60$
- B.  $g = \frac{(1.50)(50)(9.3)(2.60)}{(33)}$
- C.  $g = \frac{(1.50)(50)}{(9.3)} \times \frac{33}{2.60}$
- D.  $g = \frac{(33)(2.60)(9.3)}{50 \times 1.5}$

61. A real estate company is selling building lots of various dimensions in a new development. The price of each lot is directly proportional to its area. If a 200-foot by 200-foot parcel costs \$24,000, what is the cost of a 260-foot by 200-foot lot?
- A. \$31,200
- B. \$40,000
- C. \$40,560
- D. \$52,000

62. Two classes are planning to go on a field trip together. One class with 18 students is being joined by 6 boys and 11 girls from another class, giving an overall ratio of boys to girls on the field trip of 2 to 3. Boys made up what proportion of the original class?

- A.  $\frac{4}{9}$
- B.  $\frac{1}{2}$
- C.  $\frac{5}{9}$
- D.  $\frac{2}{3}$

63. Use the table below to answer the question that follows.

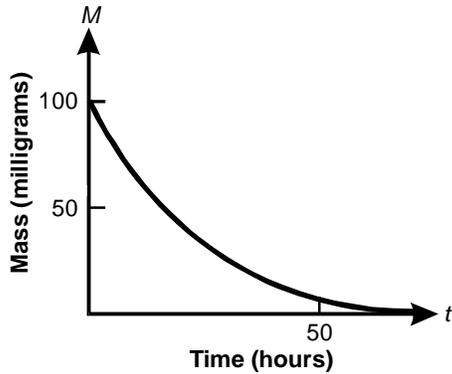
	Tomatoes	Peppers
Hot	8	8
Medium	10	6
Mild	10	4
1 batch of chili = 20 servings		

A company is planning a picnic at which chili will be served for 400 people, about 20% of whom will want hot chili, 30% will want medium chili, and 50% will want mild chili. The table above indicates the number of tomatoes and peppers needed for one batch of each type of chili. What is the ratio of tomatoes to peppers that should be purchased to ensure that the proper amount of each type of chili is available?

- A. 2 to 1
- B. 16 to 9
- C. 14 to 9
- D. 4 to 3

64. Which of the following is a factor of the equation  $3x^2 = -12x + 36$ ?
- A.  $(x + 2)$
  - B.  $(x + 3)$
  - C.  $(x + 4)$
  - D.  $(x + 6)$
65. A rectangle has an area of 375 square units and a perimeter of 80 units. If  $x$  is the length of one side of the rectangle, which of the following equations could be solved to find  $x$ ?
- A.  $x^2 - 40x - 375 = 0$
  - B.  $x^2 - 40x + 375 = 0$
  - C.  $x^2 + 40x - 375 = 0$
  - D.  $x^2 + 40x + 375 = 0$
66. A sum of \$500 is deposited in an account that bears a 6% annual rate of interest. The interest is compounded at the end of each year and added to the account. If  $N$  represents the number of years since the initial deposit, which of the following equations best models the total amount of money in the account?
- A. Amount =  $500 + 500(1.06)^N$
  - B. Amount =  $500 + 500(0.06)^N$
  - C. Amount =  $500(1.06)^N$
  - D. Amount =  $500(0.06)^N$

67. Use the graph below to answer the question that follows.



One hundred milligrams of a radioactive material are used in an experiment. The graph above shows how the mass,  $M$ , of the material varies with time. The half-life of the material is 13.2 hours. Which of the following equations best models the decay of the mass with respect to time,  $t$ , in hours?

- A.  $M(t) = 100 + 13.2t^2$
- B.  $M(t) = 100t + 13.2t^2$
- C.  $M(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{13.2}}$
- D.  $M(t) = 100\left(\frac{1}{2}\right)^{13.2t}$

68. The length of time,  $T$ , it takes to download a large file over a computer network is inversely proportional to the data transfer rate,  $D$ . It takes 48 minutes to download a large file at a rate of 56 kilobytes per second. Which of the following equations could be used to determine the time it would take to download the same file at different transfer rates?

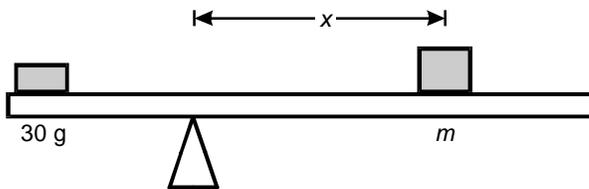
A.  $T = \frac{48D}{56}$

B.  $T = \frac{48}{56D}$

C.  $T = \frac{56}{48D}$

D.  $T = \frac{48 \cdot 56}{D}$

69. Use the diagram below to answer the question that follows.



The diagram above shows a 30-gram mass at the end of a board that is at rest on a fulcrum. The mass,  $m$ , needed to balance the 30-gram mass is inversely proportional to  $x$ , its distance from the fulcrum. The board balances when  $m = 30$  grams and  $x = 60$  centimeters. What mass  $m$  is needed to balance the board when  $x = 20$  centimeters?

- A. 40 grams  
 B. 60 grams  
 C. 90 grams  
 D. 120 grams

70. Each member of a family of four drinks 8 ounces of milk per day. If they purchase their milk from the supermarket in 1-gallon containers, how many gallons of milk are needed to get through one week?
- A. 1
  - B. 2
  - C. 3
  - D. 4
71. What is the minimum amount of paper needed to produce a label that will completely cover the curved surface of a cylindrical can with a diameter of 6 centimeters and a height of 10 centimeters?
- A. 57.68 cm<sup>2</sup>
  - B. 188.4 cm<sup>2</sup>
  - C. 244.9 cm<sup>2</sup>
  - D. 282.6 cm<sup>2</sup>
72. Which of the following temperatures is closest to 98.6° Fahrenheit, the average body temperature of a healthy human?
- A. 0° Celsius
  - B. 15° Celsius
  - C. 35° Celsius
  - D. 66° Celsius

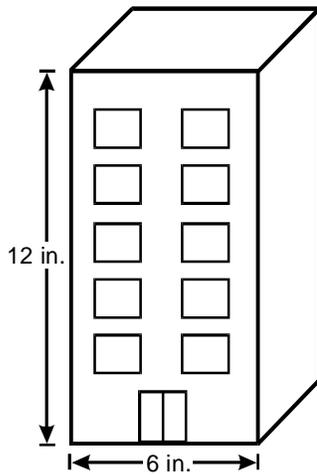
73. Use the table below to answer the question that follows.

Size	Surface Area of Ball (cm <sup>2</sup> )
Small	25
Medium	75

A manufacturer of pet supplies makes rubber balls of different sizes. Given the surface areas of the small and medium-sized balls, what is the ratio of the diameter of a medium ball to a small ball?

- A.  $\sqrt{3}:1$
- B.  $3:1$
- C.  $\sqrt[3]{5}:1$
- D.  $5:1$

74. Use the diagram below to answer the question that follows.

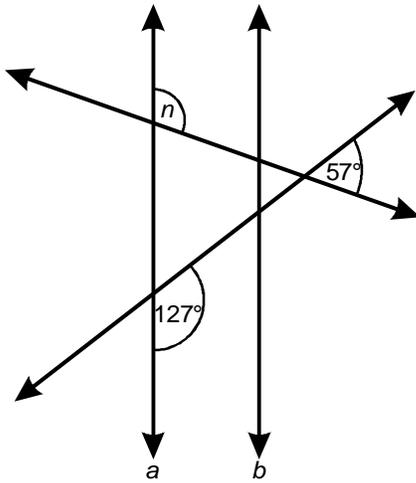


An architect has created the scale-model shown for a new building. If the front of the actual building is going to be 45 feet in width, which of the following equations can be solved to find the height,  $h$ , of the building?

- A.  $\frac{1}{0.5} = \frac{45}{h}$
- B.  $\frac{6}{45} = \frac{h}{12}$
- C.  $\frac{6}{0.5} = \frac{45}{h}$
- D.  $\frac{0.5}{1} = \frac{45}{h}$
75. Doubling the length of both the base and the height of a right triangle will cause the:
- A. area to increase by a factor of two.
- B. area to increase by a factor of four.
- C. perimeter to increase by a factor of four.
- D. perimeter to increase by a factor of eight.

76. Acceleration is a measure of how rapidly velocity is changing with respect to time. Which of the following units are most appropriate for describing the rate at which a car is accelerating?
- A.  $\frac{\text{feet}}{\text{sec.}}$
  - B.  $\frac{\text{feet}^2}{\text{sec.}}$
  - C.  $\frac{\text{feet}}{\text{sec.}^2}$
  - D.  $\frac{\text{feet}^2}{\text{sec.}^2}$
77. Given that 1.7 kilometers is approximately equal to a mile, a vehicle traveling at a rate of 60 miles per hour is moving at about how many meters per second?
- A. 10 m/s
  - B. 28 m/s
  - C. 127 m/s
  - D. 367 m/s

78. Use the diagram below to answer the question that follows.



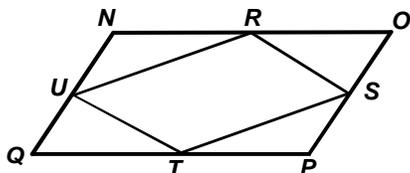
What is the measure of angle  $n$  in the diagram above?

- A.  $110^\circ$
- B.  $114^\circ$
- C.  $123^\circ$
- D.  $127^\circ$

79. Use the proof below to answer the question that follows.

**Given:**  $NOPQ$  is a parallelogram with midpoints  $R$ ,  $S$ ,  $T$ , and  $U$  on sides  $\overline{NO}$ ,  $\overline{OP}$ ,  $\overline{PQ}$ , and  $\overline{QN}$ , respectively

**Prove:**  $RSTU$  is a parallelogram



**Statements**

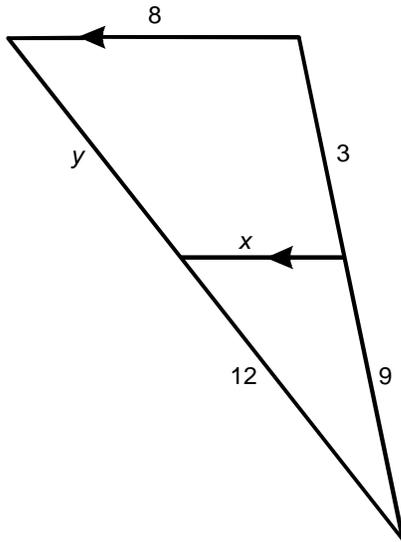
**Reasons**

- |                                                                               |                                                     |
|-------------------------------------------------------------------------------|-----------------------------------------------------|
| 1. $\angle O \cong \angle Q$<br>$\angle P \cong \angle N$                     | 1. Opposite angles of parallelograms are congruent. |
| 2. $\overline{NO} \cong \overline{PQ}$<br>$\overline{OP} \cong \overline{QN}$ | 2. Opposite sides of parallelograms are congruent.  |
| 3. $\overline{TQ} \cong \overline{RO}$<br>$\overline{UN} \cong \overline{SP}$ | 3. Definition of the midpoint                       |
| 4. $\triangle ORS \cong \triangle QTU$<br>$\triangle PST \cong \triangle NUR$ | 4. <u>?</u>                                         |
| 5. $\overline{RS} \cong \overline{TU}$<br>$\overline{ST} \cong \overline{UR}$ | 5. By corresponding parts                           |
| 6. $RSTU$ is a parallelogram                                                  | 6. Definition of a parallelogram                    |

Which of the following relationships for establishing congruence of triangles justifies Statement 4?

- A. AAS
- B. ASA
- C. SSS
- D. SAS

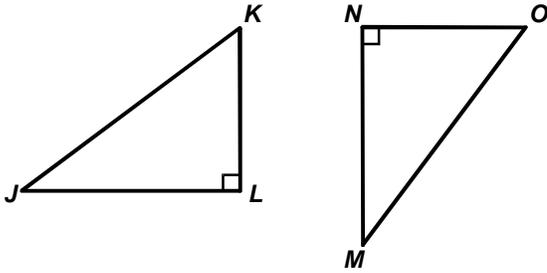
80. Use the figure below to answer the question that follows.



What is the ratio of  $x$  to  $y$  in the figure above?

- A. 2:1
- B. 3:2
- C. 3:1
- D. 4:3

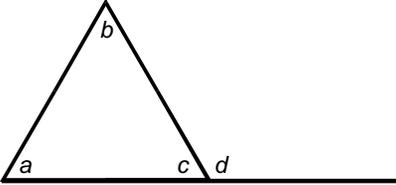
81. Use the figure below to answer the question that follows.



In the figure above, the establishment of which of the following conditions demonstrates that  $\triangle JKL$  is similar to  $\triangle MON$ ?

- A.  $\overline{KL} \cong \overline{ON}$
- B.  $\angle J + \angle K = \angle M + \angle O$
- C.  $\angle J \cong \angle M$
- D.  $\overline{JK} \cong \overline{MO}$

82. Use the figure and conjecture below to answer the question that follows.



The measure of an exterior angle of a triangle is equal to the sum of the measures of the nonadjacent interior angles.

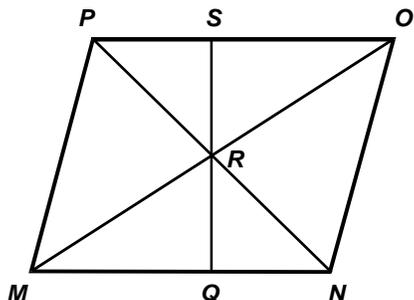
Which of the following expressions best demonstrates the validity of the conjecture?

- A.  $a + b + c = c + d$ , therefore  $a + b = d$
- B.  $a + c = d$ , therefore  $d = 180^\circ - b$
- C.  $a + b + d = c + d$ , therefore  $a + b = c$
- D.  $a + b + c = 180^\circ$ , therefore  $d = 180^\circ - (a + b)$

83. Use the proof below to answer the question that follows.

**Given:**  $MNOP$  is a parallelogram.

**Prove:**  $\overline{QR} \cong \overline{SR}$



**Statements**

1.  $\angle RMQ \cong \angle ROS$

2.  $\angle MRQ \cong \angle ORS$

3.  $\overline{MR} \cong \overline{OR}$

4.  $\triangle RMQ \cong \triangle ROS$

5.  $\overline{QR} \cong \overline{SR}$

**Reasons**

1. Alternate interior angles of two parallel lines cut by a transversal are congruent.

2. Vertical angles are congruent.

3. ?

4. ASA congruence theorem

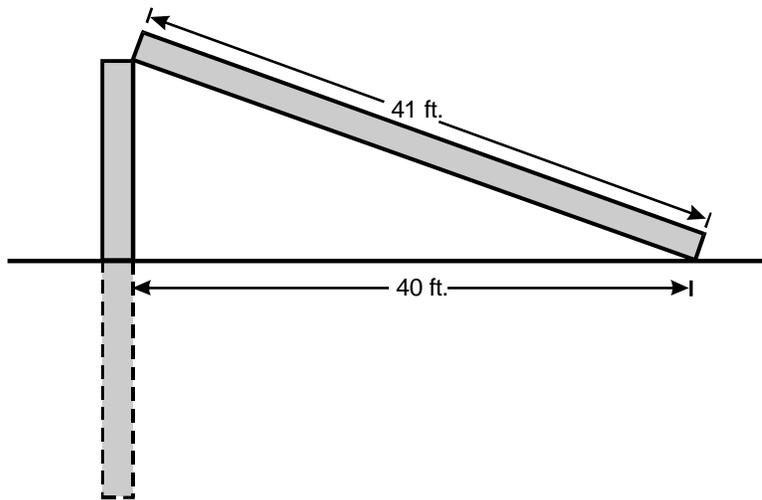
5. Corresponding parts of congruent triangles are congruent.

Which of the following reasons justifies Statement 3?

- A. Application of the Pythagorean theorem.
- B. Corresponding parts of congruent triangles are congruent.
- C. Sides opposite base angles of an isosceles triangle are congruent.
- D. Diagonals of a parallelogram bisect each other.

84. A defining characteristic of the axiomatic method when it is used to generate a system of knowledge is that proofs are based on:
- A. compass and straightedge constructions.
  - B. application of the principles of deductive logic.
  - C. arguments that are intuitively obvious.
  - D. observation of patterns and generation of conjectures.

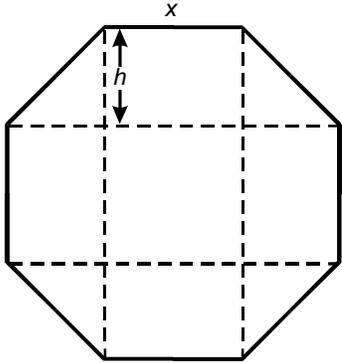
85. Use the diagram below to answer the question that follows.



For stability, 20% of the length of a utility pole was buried when it was installed. During a storm, the pole breaks and falls to the ground so that the tip of the pole is resting on the ground, as shown in the diagram above. What was the approximate total length of the utility pole before it broke?

- A. 60 feet
- B. 63 feet
- C. 80 feet
- D. 83 feet

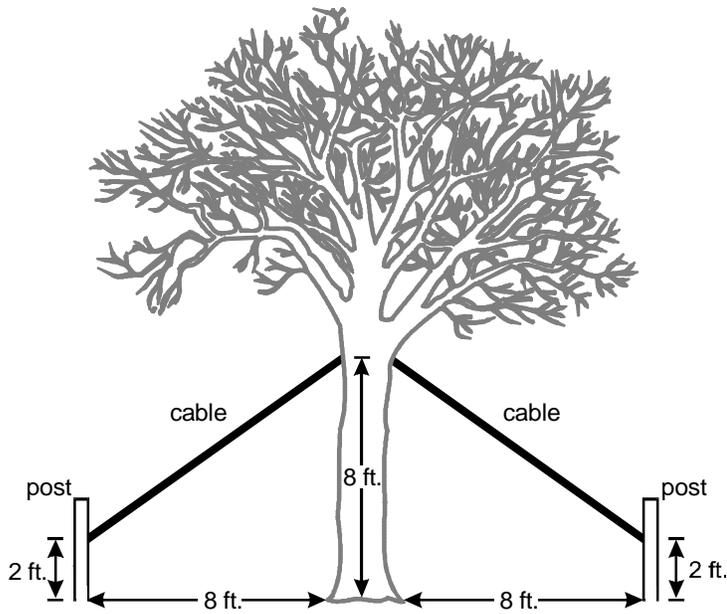
86. Use the figure below to answer the question that follows.



What is the area of the regular octagon with side length  $x$  and length  $h$  as indicated in the figure above?

- A.  $7xh$
- B.  $7x^2$
- C.  $x^2(2 + 2\sqrt{2})$
- D.  $x(1 + 3\sqrt{2})$
87. Which of the following statements is always true when comparing triangles?
- A. If the corresponding sides of two triangles are proportional in length and the included angles are equal in measure, then the triangles are similar.
- B. If the measures of two angles in a triangle are directly proportional to the measures of two angles in another triangle, then the two triangles are congruent.
- C. If the length of a side and the measure of an angle in a triangle are proportional to those of a side and an angle in another triangle, then the two triangles are similar.
- D. If two triangles share the same corresponding angles and only differ in their size, then the triangles are congruent.

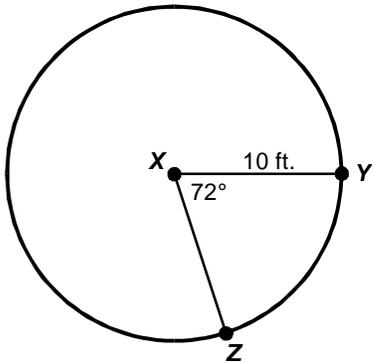
88. Use the diagram below to answer the question that follows.



The diagram depicts a tree that is supported by two separate cables attached to posts as shown in the diagram above. Each piece of cable must be 1.5 feet longer than the distance between where it attaches to the post and the tree. Assuming that the ground surface is level and that the tree is perpendicular to the ground surface, what is the total length of cable required for the project?

- A. 17 ft.
- B. 23 ft.
- C. 28 ft.
- D. 31 ft.

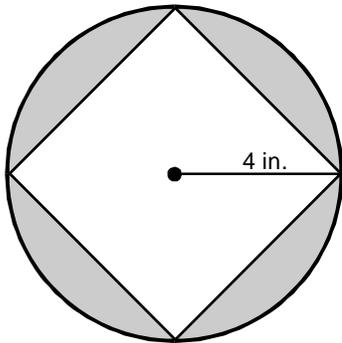
89. Use the diagram below to answer the question that follows.



For the circle in the diagram, which of the following approximations is closest to the area of sector  $XYZ$ ?

- A. 51 square feet
- B. 63 square feet
- C. 125 square feet
- D. 250 square feet

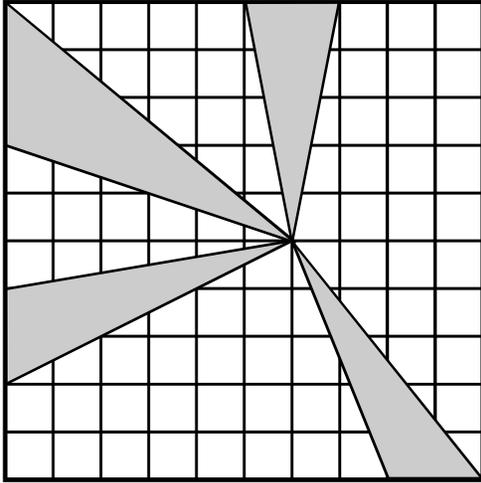
90. Use the diagram below to answer the question that follows.



The diagram above shows a square inscribed in a circle with a radius of 4 inches. Which of the following approximations is closest to the shaded area in the diagram?

- A. 24 in.<sup>2</sup>
- B. 22 in.<sup>2</sup>
- C. 20 in.<sup>2</sup>
- D. 18 in.<sup>2</sup>

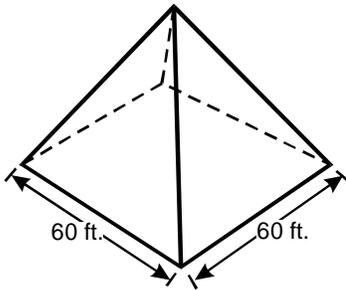
91. Use the diagram below to answer the question that follows.



The square grid measures 10 units on a side. What is the combined area of the shaded regions?

- A. 19 square units
- B. 25 square units
- C. 30 square units
- D. 50 square units

92. Use the diagram below to answer the question that follows.



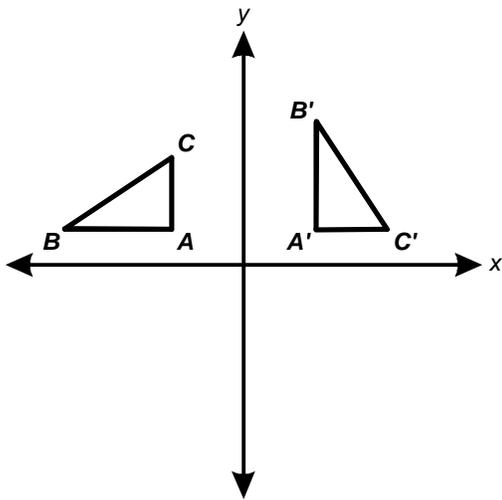
The square pyramid shown has a base that is 60 feet on a side. The vertex of the pyramid is 40 feet above its base. What is the combined surface area of the triangular faces that make up the outside of the pyramid?

- A. 1500 square feet  
 B. 4500 square feet  
 C. 4800 square feet  
 D. 6000 square feet
93. What is the area of the figure bounded by the lines in the coordinate plane whose equations are  $x = 1$ ,  $y = 3$ ,  $y = 0$ , and  $y = x - 4$ ?
- A. 9  
 B.  $10\frac{1}{2}$   
 C.  $13\frac{1}{2}$   
 D. 18

94. Quadrilateral  $ABCD$  is constructed on a set of coordinate axes with vertices at the points  $(2, 2)$ ,  $(-2, -2)$ ,  $(-8, 2)$ , and  $(-4, 7)$ . What is the area of  $ABCD$ ?

- A.  $8\sqrt{26}$  square units
- B. 42 square units
- C. 45 square units
- D.  $4\sqrt{122}$  square units

95. Use the diagram below to answer the question that follows.



Which of the following transformations would transform  $\triangle ABC$  into  $\triangle A'B'C'$  in the diagram above?

- A. a rotation around the origin
- B. a rotation around the origin and a translation
- C. a glide-reflection
- D. a glide-reflection and a translation

96. A monohedral tessellation is a tessellation composed entirely of one type of regular polygon. How many types of regular polygons are capable of forming monohedral tessellations?
- A. 3
  - B. 4
  - C. 6
  - D. 8

97. Use the diagram below to answer the question that follows.

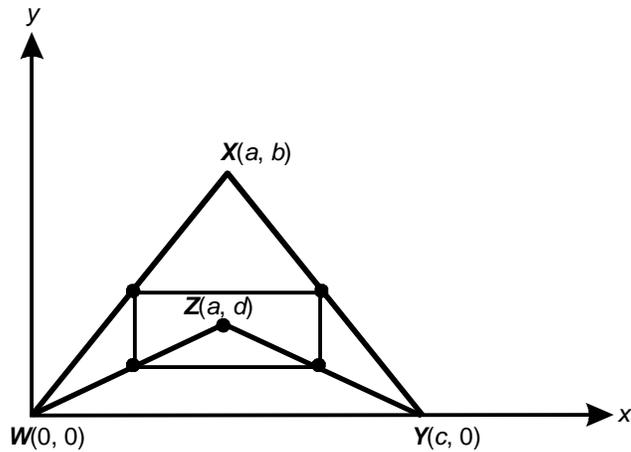
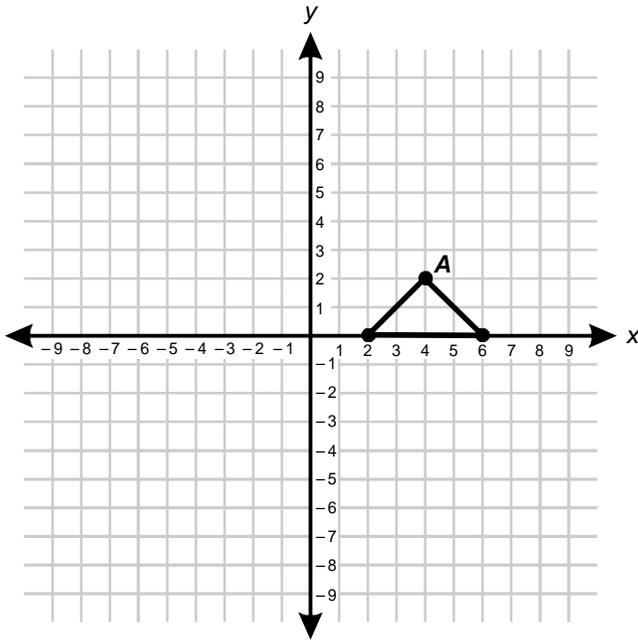


Figure  $WXYZ$  is placed on a set of coordinate axes with coordinates  $W(0, 0)$ ,  $X(a, b)$ ,  $Y(c, 0)$ , and  $Z(a, d)$ . The vertices of the rectangle are at the midpoints of  $\overline{WX}$ ,  $\overline{XY}$ ,  $\overline{YZ}$ , and  $\overline{ZW}$ . Which of the following expressions represents the area of the rectangle?

- A.  $\left(\frac{c}{2}\right)\left(\frac{b-d}{2}\right)$
- B.  $\left(\frac{c-a}{2}\right)\left(\frac{b-d}{2}\right)$
- C.  $(c)(b-d)$
- D.  $(c-a)(b-d)$

98. Use the diagram below to answer the question that follows.



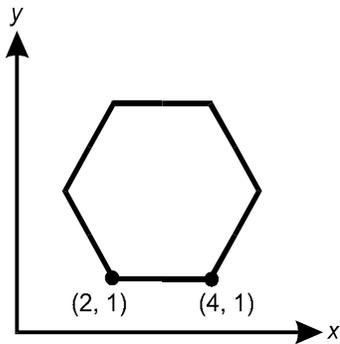
When the triangle in the diagram above is reflected across the line  $y = x$  and then across the line  $x = -1$ , what will be the coordinates of the resulting point,  $A'$ ?

- A.  $(-6, -2)$
- B.  $(-2, 6)$
- C.  $(-4, 4)$
- D.  $(2, -4)$

99. An arc is traced when point  $P(2, 4)$  is rotated  $120^\circ$  about the origin. What is the area of the circular sector formed by the arc?

- A.  $\frac{4}{3}\pi$
- B.  $\frac{20}{3}\pi$
- C.  $4\pi$
- D.  $\sqrt{20}\pi$

100. Use the diagram below to answer the question that follows.



A regular hexagon has two of its vertices at the points  $(2, 1)$  and  $(4, 1)$  on a coordinate grid, as shown in the diagram above. What is the area of the hexagon?

- A. 3
- B.  $3\sqrt{3}$
- C. 6
- D.  $6\sqrt{3}$



## DIRECTIONS FOR THE OPEN-RESPONSE ITEM ASSIGNMENTS

This section of the test consists of two open-response item assignments that appear on the following pages. You will be asked to prepare a written response of approximately 150–300 words (1–2 pages) for each assignment. You should use your time to plan, write, review, and edit your response for each assignment.

For each assignment, read the topic and directions carefully before you begin to work. Think about how you will organize your response. You may use any blank space in this test booklet to make notes, write an outline, or otherwise prepare your response.

As a whole, your response to each assignment must demonstrate an understanding of the knowledge of the field. In your response to each assignment, you are expected to demonstrate the depth of your understanding of the subject area by applying your knowledge rather than by merely reciting factual information.

Your response to each assignment will be evaluated based on the following criteria.

- **PURPOSE:** the extent to which the response achieves the purpose of the assignment
- **SUBJECT KNOWLEDGE:** appropriateness and accuracy in the application of subject knowledge
- **SUPPORT:** quality and relevance of supporting evidence
- **RATIONALE:** soundness of argument and degree of understanding of the subject area

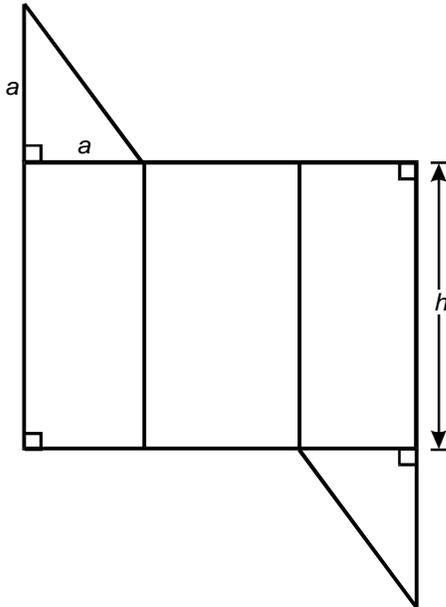
The open-response item assignments are intended to assess subject knowledge. Your responses must be communicated clearly enough to permit valid judgment of the evaluation criteria by scorers. Your responses should be written for an audience of educators in this field. The final version of each response should conform to the conventions of edited American English. Your responses should be your original work, written in your own words, and not copied or paraphrased from some other work.

Be sure to write about the assigned topics. Please write legibly. You may not use any reference materials during the test. Remember to review your work and make any changes you think will improve your responses.

Write or print your response in the space provided following the assignment.

**OPEN-RESPONSE ITEM ASSIGNMENT #1**

Use the diagram below to complete the exercise that follows.



A "net" is a two-dimensional diagram that can be cut along its edges and formed into a three-dimensional figure. Use your knowledge of measurement and geometry to analyze the characteristics of the three-dimensional figure that the net above represents. In your response, you should:

- draw a sketch of the three-dimensional figure whose net is shown above and name the figure;
- label the dimensions on your three-dimensional sketch using the variables given on the net shown above;
- use these variables to derive an equation for the volume of the three-dimensional figure; and
- use these variables to derive an equation for the surface area of the three-dimensional figure.





**OPEN-RESPONSE ITEM ASSIGNMENT #2**

Use the information below to complete the exercise that follows.

Problem 1: Simplify  $\frac{2 \cdot 11 + 4 \cdot 55}{550}$ .

Problem 2: Simplify  $\frac{a^2b^2 + a^3bc}{a^3b^4c}$ .

Use the two problems above to create a response in which you analyze how number properties and concepts apply to the solutions of problems in algebra. In your response, you should:

- solve Problem 1 and explain and justify the steps used in the solution;
- solve Problem 2 and explain and justify the steps used in the solution;
- discuss the similarities between the processes and properties used in finding the solutions to the two problems.

Be sure to show your work and explain your reasoning.





## **PRACTICE TEST RESULTS**

## PRACTICE TEST RESULTS OVERVIEW

The practice test provides valuable information regarding your preparedness for the MTEL Elementary Mathematics (53) test. In this section, you will find information and tools to help you determine your preparedness on the various sections of the test.

### Multiple-Choice Questions

A Multiple-Choice Question Answer Key Worksheet is provided to assist you in evaluating your multiple-choice responses. The worksheet contains five columns. The first column indicates the multiple-choice question number, the second column indicates the objective to which the test question was written, and the third column indicates the correct response. The remaining columns are for your use in calculating the number of multiple-choice questions you answered correctly or incorrectly.

An Evaluation Chart for the multiple-choice questions is also provided to help you assess which content covered by the test objectives may require additional study.

### Open-Response Items

Evaluation Information, Sample Responses and Analyses, as well as a Scoring Rubric are provided for these items. You may wish to refer to this information when evaluating your practice test responses.

### Total Test

Practice Test Score Calculation information is provided to help you estimate your score on the practice test. Although you cannot use this practice test to precisely predict how you might score on an official MTEL Elementary Mathematics (53) test, you may be able to determine your degree of readiness to take an MTEL test at an operational administration. No passing score has been determined for the practice test.

**MULTIPLE-CHOICE QUESTION  
ANSWER KEY WORKSHEET**

Question Number	Objective Number	Correct Response	Your Response	
			Correct?	Incorrect?
1	0001	D		
2	0001	B		
3	0001	A		
4	0001	C		
5	0001	A		
6	0001	A		
7	0002	B		
8	0002	D		
9	0002	D		
10	0002	C		
11	0002	B		
12	0002	C		
13	0003	D		
14	0003	B		
15	0003	A		
16	0003	B		
17	0003	A		
18	0003	B		
19	0004	A		
20	0004	D		
21	0004	C		
22	0004	C		
23	0004	D		
24	0004	B		
25	0005	A		
26	0005	A		
27	0005	C		
28	0005	B		
29	0005	D		
30	0005	B		
31	0005	C		
32	0006	C		
33	0006	B		
34	0006	A		

**MULTIPLE-CHOICE QUESTION  
ANSWER KEY WORKSHEET (continued)**

Question Number	Objective Number	Correct Response	Your Response	
			Correct?	Incorrect?
35	0006	D		
36	0006	A		
37	0006	A		
38	0006	D		
39	0007	D		
40	0007	A		
41	0007	D		
42	0007	B		
43	0007	A		
44	0007	C		
45	0008	C		
46	0008	B		
47	0008	D		
48	0008	C		
49	0008	D		
50	0008	C		
51	0009	A		
52	0009	B		
53	0009	D		
54	0009	A		
55	0009	C		
56	0009	A		
57	0009	B		
58	0010	B		
59	0010	D		
60	0010	A		
61	0010	A		
62	0010	A		
63	0010	B		
64	0011	D		
65	0011	B		
66	0011	C		
67	0011	C		
68	0011	D		

**MULTIPLE-CHOICE QUESTION  
ANSWER KEY WORKSHEET (continued)**

Question Number	Objective Number	Correct Response	Your Response	
			Correct?	Incorrect?
69	0011	C		
70	0012	B		
71	0012	B		
72	0012	C		
73	0012	A		
74	0012	D		
75	0012	B		
76	0012	C		
77	0012	B		
78	0013	A		
79	0013	D		
80	0013	B		
81	0013	C		
82	0013	A		
83	0013	D		
84	0013	B		
85	0014	B		
86	0014	C		
87	0014	A		
88	0014	B		
89	0014	B		
90	0014	D		
91	0014	B		
92	0014	D		
93	0015	C		
94	0015	C		
95	0015	B		
96	0015	A		
97	0015	A		
98	0015	C		
99	0015	B		
100	0015	D		

**Count the number of multiple-choice questions you answered correctly:**

\_\_\_\_\_ of 100 multiple-choice questions

## MULTIPLE-CHOICE QUESTION PRACTICE TEST EVALUATION CHART

In the evaluation chart that follows, the multiple-choice questions are arranged in numerical order and by test objective. Check your responses against the correct responses provided to determine how many questions within each objective you answered correctly.

### Subarea I: Number Sense, Operations, and Data Analysis

**Objective 0001: Understand number theory, the structure of numeration systems, and the properties of the real number system.**

1D \_\_\_\_ 2B \_\_\_\_ 3A \_\_\_\_ 4C \_\_\_\_ 5A \_\_\_\_ 6A \_\_\_\_      \_\_\_\_/6

**Objective 0002: Understand principles and operations related to integers.**

7B \_\_\_\_ 8D \_\_\_\_ 9D \_\_\_\_ 10C \_\_\_\_ 11B \_\_\_\_ 12C \_\_\_\_      \_\_\_\_/6

**Objective 0003: Understand principles and operations related to fractions.**

13D \_\_\_\_ 14B \_\_\_\_ 15A \_\_\_\_ 16B \_\_\_\_ 17A \_\_\_\_ 18B \_\_\_\_      \_\_\_\_/6

**Objective 0004: Understand principles and operations related to decimals and percents.**

19A \_\_\_\_ 20D \_\_\_\_ 21C \_\_\_\_ 22C \_\_\_\_ 23D \_\_\_\_ 24B \_\_\_\_      \_\_\_\_/6

**Objective 0005: Understand descriptive statistics and the methods used in collecting, organizing, reporting, and analyzing data.**

25A \_\_\_\_ 26A \_\_\_\_ 27C \_\_\_\_ 28B \_\_\_\_ 29D \_\_\_\_ 30B \_\_\_\_ 31C \_\_\_\_      \_\_\_\_/7

**Objective 0006: Understand the fundamental principles of probability.**

32C \_\_\_\_ 33B \_\_\_\_ 34A \_\_\_\_ 35D \_\_\_\_ 36A \_\_\_\_ 37A \_\_\_\_ 38D \_\_\_\_      \_\_\_\_/7

Subarea I (Objectives 0001–0006) Total \_\_\_\_/38

**MULTIPLE-CHOICE QUESTION  
PRACTICE TEST EVALUATION CHART (continued)**

**Subarea II: Patterns, Relations, and Algebra**

<b>Objective 0007: Understand patterns and the properties of functions and relations.</b>						
39D ____	40A ____	41D ____	42B ____	43A ____	44C ____	____/6

<b>Objective 0008: Understand how to manipulate and simplify algebraic expressions and translate problems into algebraic expressions, equations, and inequalities.</b>						
45C ____	46B ____	47D ____	48C ____	49D ____	50C ____	____/6

<b>Objective 0009: Understand properties and applications of linear relations and functions.</b>							
51A ____	52B ____	53D ____	54A ____	55C ____	56A ____	57B ____	____/7

<b>Objective 0010: Understand properties and applications of ratios and proportions.</b>						
58B ____	59D ____	60A ____	61A ____	62A ____	63B ____	____/6

<b>Objective 0011: Understand properties and applications of quadratic, exponential, polynomial, and rational functions and relations.</b>						
64D ____	65B ____	66C ____	67C ____	68D ____	69C ____	____/6

**Subarea II (Objectives 0007–0011) Total \_\_\_\_/31**

**MULTIPLE-CHOICE QUESTION  
PRACTICE TEST EVALUATION CHART (continued)**

**Subarea III: Geometry and Measurement**

**Objective 0012: Understand principles, concepts, and procedures related to measurement.**

70B \_\_\_\_ 71B \_\_\_\_ 72C \_\_\_\_ 73A \_\_\_\_ 74D \_\_\_\_ 75B \_\_\_\_ 76C \_\_\_\_ 77B \_\_\_\_ \_\_\_\_/8

**Objective 0013: Understand the principles of Euclidean geometry and use them to prove theorems.**

78A \_\_\_\_ 79D \_\_\_\_ 80B \_\_\_\_ 81C \_\_\_\_ 82A \_\_\_\_ 83D \_\_\_\_ 84B \_\_\_\_ 85B \_\_\_\_ \_\_\_\_/8

**Objective 0014: Apply Euclidean geometry to analyze the characteristics and properties of two- and three-dimensional shapes.**

86C \_\_\_\_ 87A \_\_\_\_ 88B \_\_\_\_ 89B \_\_\_\_ 90D \_\_\_\_ 91B \_\_\_\_ 92D \_\_\_\_ 93C \_\_\_\_ \_\_\_\_/8

**Objective 0015: Understand the principles and properties of coordinate and transformational geometry and use them to prove theorems.**

94C \_\_\_\_ 95B \_\_\_\_ 96A \_\_\_\_ 97A \_\_\_\_ 98C \_\_\_\_ 99B \_\_\_\_ 100D \_\_\_\_ \_\_\_\_/7

**Subarea III (Objectives 0012–0015) Total \_\_\_\_/23**

## OPEN-RESPONSE ITEM EVALUATION INFORMATION

### How Open-Response Items Are Scored

Open-response items are scored through a process called focused holistic scoring. Scorers judge the overall effectiveness of the response rather than individual aspects considered in isolation. Scorer judgments are based on the quality of the response, not on length or neatness. Responses must be long enough to cover the topic adequately and scorers must be able to read what is written.

### How to Evaluate Your Practice Responses

On the following pages, you will find two "strong" and two "weak" sample responses. PLEASE DO NOT REVIEW THE SAMPLE RESPONSES UNTIL AFTER YOU HAVE WRITTEN YOUR OWN RESPONSE. When you do review the two "strong" and "weak" sample responses and analyses included here, please note the following points:

- ✓ For the purposes of the practice test, responses are identified as "strong" or "weak" rather than given a score point of 1–4.
- ✓ The responses identified as "strong" may contain flaws; however, these responses do demonstrate the performance characteristics of a "strong response."
- ✓ The two "strong" responses demonstrate the examinees' appropriate understanding and application of the subject matter knowledge. However, these responses do not necessarily reflect the full range of "correct answers" that would demonstrate an understanding of the subject matter.
- ✓ The "Analysis" accompanying each "strong" and "weak" response discusses the main attributes of the responses, but does not identify all flaws or strengths that may be present.

Compare your practice responses to the [Sample Responses](#) to determine whether your responses are more similar to the strong or weak responses. Also review the [Analyses](#) on those pages and the [Scoring Rubric](#) to help you better understand the characteristics of strong and weak responses. This evaluation will help you identify specific problems or weaknesses in your practice responses. Further information on scoring can be found in the [Test Information Booklet](#) and [Faculty Guide](#) at [www.mtel.nesinc.com](http://www.mtel.nesinc.com) and at [www.doe.mass.edu/mtel](http://www.doe.mass.edu/mtel); select "FAQ," then "After the Test."

**OPEN-RESPONSE ITEM  
SCORING RUBRIC, SAMPLE RESPONSES, AND ANALYSES**

Massachusetts Tests for Educator Licensure®  
**SCORING RUBRIC FOR SUBJECT TESTS**

**Performance Characteristics:**

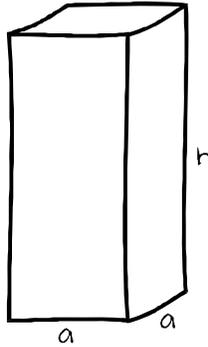
Purpose	The extent to which the response achieves the purpose of the assignment.
Subject Matter Knowledge	Accuracy and appropriateness in the application of subject matter knowledge.
Support	Quality and relevance of supporting details.
Rationale	Soundness of argument and degree of understanding of the subject matter.

**Scoring Scale:**

Score Point	Score Point Description
<b>4</b>	<p><b>The "4" response reflects a thorough knowledge and understanding of the subject matter.</b></p> <ul style="list-style-type: none"> <li>• The purpose of the assignment is fully achieved.</li> <li>• There is a substantial, accurate, and appropriate application of subject matter knowledge.</li> <li>• The supporting evidence is sound; there are high-quality, relevant examples.</li> <li>• The response reflects an ably reasoned, comprehensive understanding of the topic.</li> </ul>
<b>3</b>	<p><b>The "3" response reflects an adequate knowledge and understanding of the subject matter.</b></p> <ul style="list-style-type: none"> <li>• The purpose of the assignment is largely achieved.</li> <li>• There is a generally accurate and appropriate application of subject matter knowledge.</li> <li>• The supporting evidence is adequate; there are some acceptable, relevant examples.</li> <li>• The response reflects an adequately reasoned understanding of the topic.</li> </ul>
<b>2</b>	<p><b>The "2" response reflects a limited knowledge and understanding of the subject matter.</b></p> <ul style="list-style-type: none"> <li>• The purpose of the assignment is partially achieved.</li> <li>• There is a limited, possibly inaccurate or inappropriate, application of subject matter knowledge.</li> <li>• The supporting evidence is limited; there are few relevant examples.</li> <li>• The response reflects a limited, poorly reasoned understanding of the topic.</li> </ul>
<b>1</b>	<p><b>The "1" response reflects a weak knowledge and understanding of the subject matter.</b></p> <ul style="list-style-type: none"> <li>• The purpose of the assignment is not achieved.</li> <li>• There is little or no appropriate or accurate application of subject matter knowledge.</li> <li>• The supporting evidence, if present, is weak; there are few or no relevant examples.</li> <li>• The response reflects little or no reasoning about or understanding of the topic.</li> </ul>
<b>U</b>	<b>The response is unrelated to the assigned topic, illegible, primarily in a language other than English, not of sufficient length to score, or merely a repetition of the assignment.</b>
<b>B</b>	<b>There is no response to the assignment.</b>

**FIRST SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE  
ITEM ASSIGNMENT #1**

Draw a sketch of the three-dimensional figure whose net is shown above and name the figure.



- The two triangles connect to make a square, so this would be a box with a square bottom and rectangles for sides.

Label the dimensions on your three-dimensional sketch using the variables given on the net shown above.

- see graphic

Use these variables to derive an equation for the volume of the three-dimensional figure.

- $V = \text{base} \cdot \text{height} = a^2h$

Use these variables to derive an equation for the surface area of the three-dimensional figure.

- $A = 2(\text{base}) + 4(\text{side}) = 2a + 4h$

## ANALYSIS FOR FIRST WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

*This is an example of a weak response because it is characterized by the following:*

**Purpose:** The candidate partially achieves the purpose of the assignment. The sketch of the three-dimensional figure is incorrect. The derived formulas for the volume and surface area suggest a basic understanding of these concepts, but are applied carelessly.

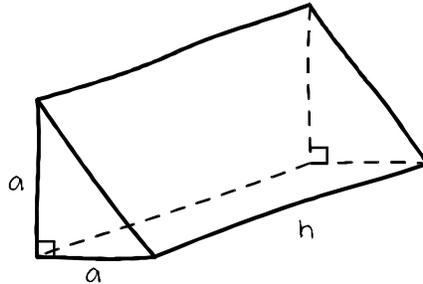
**Subject Matter Knowledge:** The candidate's representation of the three-dimensional figure implies a limited understanding of the spatial relationships in two- and three-dimensional geometry. The formulas for the volume and the surface area of a rectangular prism, " $V = \text{base} \cdot \text{height}$ " and " $A = 2(\text{base}) + 4(\text{side}) = 2a + 4h$ ," are ambiguously worded, and there is an error in the formula for surface area. The formulas do demonstrate that the candidate has some understanding of the geometric properties of three-dimensional figures.

**Support:** The candidate draws an incorrect three-dimensional figure rather than the one the assignment requires, and the formulas for volume and area are incorrect.

**Rationale:** The candidate's initial misconception weakens the overall response, but even if the formula derivations had been done correctly, the result would demonstrate only minimal understanding of the geometric concepts involved in the assignment.

**SECOND SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE  
ITEM ASSIGNMENT #1**

Draw a sketch of the three-dimensional figure whose net is shown above and name the figure.



- Triangular prism

Label the dimensions on your three-dimensional sketch using the variables given on the net shown above.

- see graphic

Use these variables to derive an equation for the volume of the three-dimensional figure.

- $V = l + w + h$   
 $= a + a + h = 2a + h$

Use these variables to derive an equation for the surface area of the three-dimensional figure.

- $A_{\text{triangle}} = \frac{1}{2}bh$   $A_{\text{rectangle}} = lw$
- $A_{\text{figure}} = 2(\frac{1}{2}ah) + 3ah = ah + 3ah = 4ah$

## ANALYSIS FOR SECOND WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

*This is an example of a weak response because it is characterized by the following:*

**Purpose:** The purpose of the assignment is only minimally achieved. Though the candidate correctly sketches the three-dimensional figure as a triangular prism from the given net, calculation of the surface area and volume is incorrect and suggests conceptual confusion.

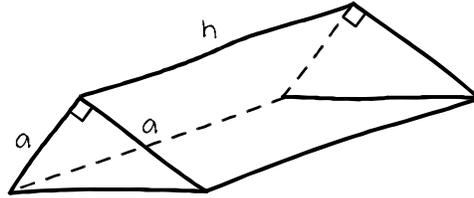
**Subject Matter Knowledge:** The candidate demonstrates the ability to adapt spatially from a two-dimensional net to the correct three-dimensional figure. However, inappropriate formulas are used to find the volume and surface area of the figure. The formula  $V = l + w + h$  cannot be associated with a triangular prism and indicates a lack of understanding of the concept of volume. The candidate's analysis of the surface area of the figure is overly simplified. The figure does contain triangles and rectangles and the candidate correctly recalls those area formulas. The candidate uses the  $h$  from the general formula for the area of a triangle as the height in the calculations instead of using the actual height of the figure. The dimensions for all three rectangles are assigned the same values.

**Support:** The organization of the response is reasonable, although the content is generally weak or incorrect. The candidate incorrectly analyzes the triangular prism by applying an incorrect formula for the volume of a rectangular prism. The decomposition of the figure with respect to its surface area defines the dimensions incorrectly by giving all three rectangular sides the same length and width.

**Rationale:** While the candidate exhibits good spatial skills, the response indicates limited understanding of how the formulas for volume and surface area apply to specific geometric figures.

**FIRST SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE  
ITEM ASSIGNMENT #1**

Draw a sketch of the three-dimensional figure whose net is shown above and name the figure.



- This is a right triangular prism, i.e., a prism with bases that are right triangles.

Label the dimensions on your three-dimensional sketch using the variables given on the net shown above.

- see graphic

Use these variables to derive an equation for the volume of the three-dimensional figure.

- The volume of a prism is calculated as the product of the area of the base of the prism and its height. The area of a right triangle is calculated as one half the product of its base and its height. In this case.

$$V = (\text{area of base triangle}) \cdot (\text{height of prism})$$

$$= \frac{1}{2}(a \times a)h = \frac{1}{2}a^2h$$

Use these variables to derive an equation for the surface area of the three-dimensional figure.

- The surface area, SA, will be the sum of the areas of the two triangular bases, the two rectangular sides, and the rectangular base.

$$SA = (2) \cdot (\text{area of base triangle}) + (2) \cdot (\text{area of lateral rectangle}) + (\text{area of base rectangle})$$

The area of a rectangle is calculated as the product of its length and its width. The width of the base rectangle is the hypotenuse of the right triangle.

$$= (2 \times \frac{1}{2}a^2) + (2 \times ah) + (\text{hypotenuse of base triangle} \cdot h)$$

By the Pythagorean theorem, the hypotenuse of the base triangle equals

$$\sqrt{a^2 + a^2} = a\sqrt{2}$$

$$SA = a^2 + 2ah + ah\sqrt{2} = a^2 + ah(2 + \sqrt{2}) \text{ square units}$$

### ANALYSIS FOR FIRST STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

*This is an example of a strong response because it is characterized by the following:*

**Purpose:** The candidate's response completely achieves the purpose of the assignment. The solution is clearly, accurately, and thoroughly presented.

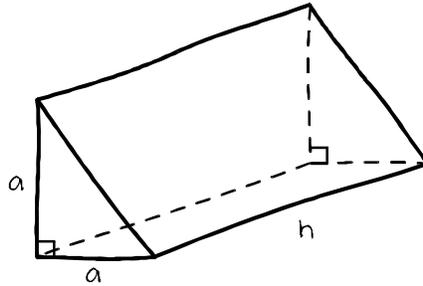
**Subject Matter Knowledge:** The candidate shows a sound knowledge of geometric concepts, including the visualization of shapes and the application of formulas. The figure is correctly named and drawn. The candidate accurately applies general formulas for volume and surface area to a specific figure, and exhibits competency in algebra by correctly simplifying the expression representing the surface area. Overall, the response reflects a substantial and accurate application of subject matter knowledge.

**Support:** Including general formulas for volume and surface area in the response results in a clear and logical explanation of what the specific formulas mean in the context of the problem. The relationship between the decomposition of the figure and the derivation of the specific formula for calculation of the surface area is evident. The verbal descriptions and the numerical example result in a thorough and well-designed response, even though the candidate could have been clearer in the use of the terms "base" and "lateral."

**Rationale:** The subject matter content and the clear organization of the candidate's response indicate a sound understanding of the geometric concepts addressed in the assignment.

**SECOND SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE  
ITEM ASSIGNMENT #1**

Draw a sketch of the three-dimensional figure whose net is shown above and name the figure.



- Triangular prism

Label the dimensions on your three-dimensional sketch using the variables given on the net shown above.

- see graphic

Use these variables to derive an equation for the volume of the three-dimensional figure.

- $V = \frac{1}{2}(a)(a)h = \frac{1}{2}a^2h$

Use these variables to derive an equation for the surface area of the three-dimensional figure.

- $SA = 2 \cdot \text{area of triangle} + 3 \cdot \text{area of rectangle}$   
 $= 2\left(\frac{1}{2}a^2h\right) + 2a + ah\sqrt{2} = a^2 + 2ah + ah\sqrt{2}$

## ANALYSIS FOR SECOND STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

*This is an example of a strong response because it is characterized by the following:*

**Purpose:** The candidate's solution responds to all parts of the assignment. Each part of the response is clear and accurate. The candidate achieves the purpose of the assignment.

**Subject Matter Knowledge:** The candidate shows competence in the geometric concepts the assignment requires. The net is recognized and the three-dimensional figure is accurately drawn and labeled, though the text describes it only as a triangular, rather than a right triangular, prism. The formulas for the volume and surface area of the figure are correctly derived.

**Support:** Overall, this is a strong response. The candidate recognizes and applies the correct formulas for calculating the volume and surface area of the shape. The response would have been improved by including in the figure the dimensions of the third rectangle used to find the surface area. The response also would have benefited from a description of the calculations required. The  $\sqrt{2}$  in the expression implies application of the Pythagorean theorem, but the formula presents the areas of the three rectangular faces as though they are equivalent. They are not equivalent because the base of the prism has the hypotenuse of the right triangle as its width. Nonetheless, while the language is imprecise, the correct values are used in the calculation and the analysis of the problem is competent.

**Rationale:** The response reflects good geometry knowledge and skills, and the organization is strong. The presentation of the solution to the assignment flows logically but not easily: a stronger response might have included more detail and intermediate steps.

**FIRST SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE  
ITEM ASSIGNMENT #2**

Solve Problem 1 and explain and justify the steps used in the solution.

$$\begin{aligned} \bullet \quad \frac{2 \cdot 11 + 4 \cdot 55}{550} &= \frac{22 + 220}{550} && \text{Multiply.} \\ &= \frac{242}{550} && \text{Add.} \\ &= \frac{121}{275} && \text{Reduce by 2.} \\ &= \frac{11}{25} && \text{Reduce by 11.} \end{aligned}$$

Solve Problem 2 and explain and justify the steps used in the solution.

$$\begin{aligned} \bullet \quad \frac{a^2b^2 + a^3bc}{a^3b^4c} &= \frac{a^2b^2}{a^3b^4c} + \frac{a^3bc}{a^3b^4c} && \text{Put each term in the numerator over the} \\ &&& \text{denominator.} \\ &= \frac{a^2b^2}{a^3b^{4+2}c} + \frac{a^3bc}{a^3b^{4+3}c} && \text{Cancel factors in each fraction.} \\ &= \frac{1}{ab^2c} + \frac{1}{b^3} && \text{Reduce each fraction.} \end{aligned}$$

Discuss the similarities between the processes and properties used in finding the solutions to the two problems.

The methods are different because in Problem 1 the numbers can be added first and then reduced. In Problem 2 they can't be added first because they involve unlike terms, but splitting the numerator allows the fractions to be reduced separately. The problems are similar because they both involve first using addition and then reducing fractions.

## ANALYSIS FOR FIRST WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

*This is an example of a weak response because it is characterized by the following:*

**Purpose:** The candidate's response only partially fulfills the goal of the assignment. The solutions to both problems are basically correct, although the solution in Problem 2 doesn't progress to the appropriate final step. The discussion of processes and properties is consistent with the methods employed in the responses but does not reflect any recognition of the general similarities between the two problems. The number properties that justify the procedures are not mentioned.

**Subject Matter Knowledge:** The solutions to both problems are correct, but are limited applications of the skill sets needed for performing number operations and manipulating algebraic fractions. The solution to Problem 2,  $\frac{1}{ab^2c} + \frac{1}{b^3}$ , should be presented as a single fraction.

**Support:** In both solutions the step-by-step justifications provided are descriptions of the processes employed. However, the descriptions fail to address or show any awareness of number properties or how they can be used to justify the steps in the solution. This failure to reference any number properties as justifications for the steps indicates a limited understanding of the general concepts that support both solution processes. The candidate's approach to solving the algebra problem is accurate but is not parallel with the number problem solution, so the inherent similarity between the two problems is not clearly evident.

**Rationale:** Overall, the response shows adequate mathematical skills in solving the problems, but does not demonstrate a full understanding of how fundamental number properties explain the similarity of the steps taken in solving both problems. As a result, the connections between arithmetic and algebra are not addressed adequately.

## SECOND SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #2

Solve Problem 1 and explain and justify the steps used in the solution.

$$\bullet \frac{2 \cdot 11 + 4 \cdot 55}{550} = \frac{22 + 220}{550} = \frac{242}{550} = \frac{121}{275}$$

Use order of operations and reduce by 2.

Solve Problem 2 and explain and justify the steps used in the solution.

$$\bullet \frac{a^2b^2 + a^3bc}{a^3b^4c} = \frac{a^2b^2 + a^3bc}{a^3b^4c} = \frac{a^2}{b} \quad \text{Cancel as much as possible because terms in numerator cannot be combined.}$$

Discuss the similarities between the processes and properties used in finding the solutions to the two problems.

The problems are similar because both involve reducing fractions by canceling. The methods are different because in Problem 1 the numbers in the numerator can be added and in Problem 2 they can't.

## ANALYSIS FOR SECOND WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

*This is an example of a weak response because it is characterized by the following:*

**Purpose:** The candidate demonstrates minimal understanding of the general purpose of the problem. The steps taken to solve Problem 1 are correct, but the solution is unfinished. In Problem 2, the steps taken are incorrect and thus the solution is incorrect. In neither case does the candidate adequately provide number properties to justify the steps taken to solve the problem.

**Subject Matter Knowledge:** The candidate's response reveals weaknesses and gaps in subject matter knowledge. While the steps taken to solve Problem 1 are correct, the solution is incomplete in that  $\frac{121}{275}$  can be simplified further to  $\frac{11}{25}$ . The justification of the steps merely cites the order of operations, a procedure, without referencing the specific number properties that underlie it. In Problem 2, the simultaneous canceling of the  $a^3$  in the second term of the numerator and the  $a^3$  in the denominator is a common error that suggests the candidate has a weak grasp of fundamental concepts governing the manipulation of common factors in polynomials and fractions. Canceling factors is necessary to solve Problem 2, but not in the manner shown. The discussion of the similarities between the processes and properties applied in the two solutions does not mention the application of any number properties, suggesting that knowledge of the properties is lacking or weak.

**Support:** The candidate's organization of the solution to both problems is poor. One goal of the assignment is to justify the steps taken in solving the problems. The form of the solution as presented does not support the

systematic provision of sequential, step-by-step justifications. No number properties are cited to justify the operational steps taken.

**Rationale:** The response to Problem 1 indicates some basic number skills. However, the incompleteness of the solution and the mishandling of the reduction of terms in the algebraic fraction in Problem 2 suggest only basic proficiency in the skill sets needed for manipulating rational numbers. This conclusion is supported further by the candidate's mishandling of the reduction of terms in the algebraic fraction in Problem 2. Overall, the faulty processes used in Problem 2 and the absence of any reference to the basic properties of numbers in both problem solutions demonstrate only minimal understanding of the assigned task.

**FIRST SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE  
ITEM ASSIGNMENT #2**

Solve Problem 1 and explain and justify the steps used in the solution.

- $$\frac{2 \cdot 11 + 4 \cdot 55}{550} = \frac{2 \cdot 11 + 2 \cdot 2 \cdot 5 \cdot 11}{2 \cdot 5 \cdot 5 \cdot 11}$$

Write each quantity as a product of primes.

$$= \frac{2 \cdot 11(1 + 2 \cdot 5)}{2 \cdot 11(5 \cdot 5)}$$

Apply the distributive and commutative properties.

$$= 1 \cdot \frac{1+10}{25}$$

Cancel since a number divided by itself is 1.

$$= \frac{11}{25}$$

One is the multiplicative identity.

Solve Problem 2 and explain and justify the steps used in the solution.

- $$\frac{a^2b^2 + a^3bc}{a^3b^4c} = \frac{a \cdot a \cdot b \cdot b + a \cdot a \cdot a \cdot b \cdot c}{a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot c}$$

Write each quantity as a product of primes.

$$= \frac{a \cdot a \cdot b(b + ac)}{a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot c}$$

Apply the distributive property.

$$= 1 \cdot \frac{b + ac}{ab^3c}$$

Cancel since a number divided by itself is 1.

$$= \frac{b + ac}{ab^3c}$$

One is the multiplicative identity.

Discuss the similarities between the processes and properties used in finding the solutions to the two problems.

- Problem 1 could be solved by computing  $\frac{242}{550}$  and reducing the fraction. However, using prime factors and the distributive property to factor out the greatest common factor shows how the properties of numbers justify the arithmetic procedure. This also makes it easy to see how the same properties can be used when using variables instead of numbers. Doing the problems together clarifies and justifies the algebraic process used in Problem 2.

## ANALYSIS FOR FIRST STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

*This is an example of a strong response because it is characterized by the following:*

**Purpose:** All aspects of the assignment have been fully achieved. The solutions to both problems are correct and accurate. The candidate clearly comprehended all aspects of the task and approached the solution of each in a way that supported the task. The candidate's final analysis clearly demonstrates the connection between number and algebraic concepts.

**Subject Matter Knowledge:** The detailed solutions to both problems accurately and appropriately apply the concepts of prime factors, the distributive property, the identity property of 1, and the multiplicative property of 1 in each of the four steps. This indicates a thorough understanding of the task and the concepts needed to complete it successfully.

**Support:** The candidate's solutions to both problems are well thought out and organized. Each of the steps used in the solutions is clearly and accurately stated and applied. The candidate uses the same number of steps to solve each problem and also uses identical justifications, referencing properties as well as processes for those justifications. The parallelism in both structure and terminology in the two solutions clearly demonstrates their similarities and supports the generalization of the processes and properties to other algebra problems.

**Rationale:** The candidate shows mastery of the skills necessary to perform basic algebraic operations. The response is strengthened by the candidate's statement that "Problem 1 could be solved by computing  $\frac{242}{550}$  and reducing the fraction." The approach actually used by the candidate led to a far deeper analysis than a simple arithmetic approach would have, and so demonstrates an excellent grasp of the intent of the problem. The organization of the solutions and their justifications reflects a comprehensive understanding of number and algebraic concepts and of the connections between arithmetic and algebra.

## SECOND SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #2

Solve Problem 1 and explain and justify the steps used in the solution.

$$\bullet \quad \frac{2 \cdot 11 + 4 \cdot 55}{550} = \frac{2 \cdot 11(1 + 2 \cdot 5)}{2 \cdot 11(25)} \quad \text{Take out GCF.}$$

$$= \frac{1 + 10}{25} \quad \text{Cancel } 2 \cdot 11 \text{ in the numerator and denominator.}$$

Solve Problem 2 and explain and justify the steps used in the solution.

$$\bullet \quad \frac{a^2b^2 + a^3bc}{a^3b^4c} = \frac{a^2 \cdot b(b + ac)}{a^2 \cdot b(ab^3c)} \quad \text{Take out GCF.}$$

$$= \frac{b + ac}{ab^3c} \quad \text{Cancel } a^2b \text{ in the numerator and denominator.}$$

Discuss the similarities between the processes and properties used in finding the solutions to the two problems.

- Both problems apply the same procedures, factoring, means of finding the GCF, and methods of cancelling terms. These processes and properties are the same in the numerator as they are in the denominator.

## ANALYSIS FOR SECOND STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

*This is an example of a strong response because it is characterized by the following:*

**Purpose:** The candidate's response largely achieves the purpose of the assignment. The solutions to both problems are correct and accurate, and the application of similar techniques and number properties in both cases is recognized. However, the role of number properties and the underlying connections between arithmetic and algebra could have been analyzed in greater detail.

**Subject Matter Knowledge:** The solutions to both problems are correct and reflect strong algebra skills. The factoring out of  $2 \cdot 11$  in the first step of Problem 1 shows good recognition of and facility with the handling of algebraic expressions, and the fraction is reduced to its simplest form. The manipulations in Problem 2 are accurately done and clearly presented. The candidate is familiar with and knows how to apply the underlying number properties (e.g., distributive law, multiplicative identity) that justify the procedures employed. The use of appropriate terminology (e.g., GCF) also indicates the candidate's familiarity with number properties and procedures like factorization.

**Support:** The parallel organization of the solutions and the referencing of the same procedures illustrate the similarity of the two problem solutions. The response would have benefited from the candidate's identification of the application of the distributive property in both solutions (and the commutative property in Problem 1). The candidate is clearly familiar with the number properties and applies them appropriately. However, because steps in the solution have been combined, the justifications for algebraic procedures are not obvious. The response would have been stronger if applications of specific number properties had been mentioned.

**Rationale:** The candidate displays strong algebra skills and succeeds in providing accurate solutions to the two problems. The goal of the assignment—justifying the mathematical steps in the two solutions with number properties—would have been addressed more fully if the candidate had shown how the steps provide logical connections between procedures of arithmetic and algebra.

## PRACTICE TEST SCORE CALCULATION

The practice test score calculation is provided so that you may better gauge your performance and degree of readiness to take an MTEL test at an operational administration. Although the results of this practice test may be used as one indicator of potential strengths and weaknesses in your knowledge of the content on the official test, it is not possible to predict precisely how you might score on an official MTEL test.

The Sample Responses and Analyses for the open-response items may help you determine whether your responses are more similar to the strong or weak samples. The Scoring Rubric can also assist in estimating a score for your open responses. You may also wish to ask a mentor or teacher to help evaluate your responses to the open-response questions prior to calculating your total estimated score.

### How to Calculate Your Practice Test Score

Review the directions in the sample below and then use the blank practice test score calculation worksheet on the following page to calculate your estimated score.

#### SAMPLE

<b>Multiple-Choice Section</b>	
Enter the total number of multiple-choice questions you answered correctly:	<u>71</u>
Use Table 1 below to convert that number to the score and write your score in <b>Box A</b> :	<b>A:</b> <input type="text" value="195"/>

<b>Open-Response Section</b>	
Enter the number of points (1 to 4) for your first open-response question:	<u>3</u>
Enter the number of points (1 to 4) for your second open-response question:	<u>3</u>
	=====
Add those two numbers (number of open-response question points):	6
Use Table 2 below to convert that number to the score and write your score in <b>Box B</b> :	<b>B:</b> <input type="text" value="52"/>

<b>Total Practice Test Score (Estimated MTEL Score)</b>	
Add the numbers in <b>Boxes A and B</b> for an estimate of your MTEL score:	<b>A + B =</b> <input type="text" value="247"/>

Practice Test Score Calculation Worksheet: Elementary Mathematics

Table 1:

Number of Multiple-Choice Questions Correct	Estimated MTEL Score		Number of Multiple-Choice Questions Correct	Estimated MTEL Score
0 to 25	113		61 to 65	179
26 to 30	121		66 to 70	187
31 to 35	129		71 to 75	195
36 to 40	137		76 to 80	204
41 to 45	146		81 to 85	212
46 to 50	154		86 to 90	220
51 to 55	162		91 to 95	228
56 to 60	170		96 to 100	237

Table 2:

Number of Open-Response Question Points	Estimated MTEL Score
2	36
3	40
4	44
5	48
6	52
7	56
8	60

Print the form below to calculate your estimated practice test score.

**Multiple-Choice Section**

Enter the total number of multiple-choice questions you answered correctly:

Use Table 1 above to convert that number to the score and write your score in **Box A**:

A:

**Open-Response Section**

Enter the number of points (1 to 4) for your first open-response question:

Enter the number of points (1 to 4) for your second open-response question:

=====

Add those two numbers (number of open-response question points):

Use Table 2 above to convert that number to the score and write your score in **Box B**:

B:

**Total Practice Test Score (Estimated MTEL Score)**

Add the numbers in **Boxes A and B** for an estimate of your MTEL score:

A + B =