**Massachusetts
Tests for Educator Licensure® (MTEL®)**

MTEL®

General Curriculum (103/203)

PRACTICE TEST

BOOKLET 2
Mathematics Subtest
(203)
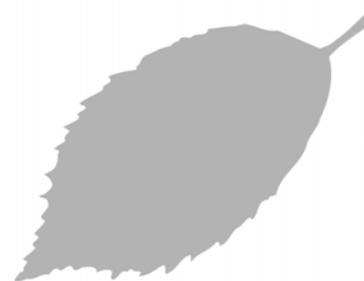
*Multiple-Choice Question Analyses are provided in a separate appendix document.
See the MTEL web site for more information.*

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INTRODUCTION

This document is a printable version of the Massachusetts Tests for Educator Licensure® (MTEL®) General Curriculum (03) Online Practice Test.

The MTEL General Curriculum test consists of a multi-subject subtest and a mathematics subtest. To pass the General Curriculum test, you must pass both the multi-subject and mathematics subtests.

This practice test for the General Curriculum mathematics subtest is a sample test consisting of 45 multiple-choice questions and 1 open-response item assignment.

To assist you in recording and evaluating your responses on the practice test, a <u>Multiple-Choice Answer Sheet</u>, an <u>Answer Key Worksheet</u>, an <u>Evaluation Chart</u> by test objective, and <u>Analyses</u> are included for the multiple-choice questions. A blank <u>Response Sheet</u>, <u>Evaluation Information</u>, and <u>Sample Responses and Analyses</u>, as well as a <u>Scoring Rubric</u>, are included for the open-response item. Lastly, there is a <u>Practice Test Score Calculation Worksheet</u>.

PURPOSE OF THE PRACTICE TEST

The practice test is designed to provide an additional resource to help you effectively prepare for the MTEL General Curriculum (03) test. The primary purpose of the practice test is to help you become familiar with the structure and content of the test. It is also intended to help you identify areas in which to focus your studies. Education faculty and administrators of teacher preparation programs may also find this practice test useful as they help students prepare for the official test.

TAKING THE PRACTICE TEST

In order to maximize the benefits of the practice test, it is recommended that you take this test under conditions similar to the conditions under which the official MTEL tests are administered. Try to take the practice test in a quiet atmosphere with few interruptions and limit yourself to the four-hour time period* allotted for the official test administration. You will find your results to be more useful if you refer to the answer key and/or multiple-choice question analyses only after you have completed the practice test.

INCORPORATING THE PRACTICE TEST IN YOUR STUDY PLAN

Although the primary means of preparing for the MTEL is your college education, adequate preparation prior to taking or retaking the MTEL test is strongly recommended. How much preparation and study you need depends on how comfortable and knowledgeable you are with the content of the test.

The first step in preparing to take the MTEL is to identify what information the test will address by reviewing the objectives for your field. A complete, up-to-date list of the <u>Test Objectives</u> is included in the <u>Test Information Booklet</u> for each test field. The test objectives are the core of the testing program and a helpful study tool. Before taking or retaking the official test, focus your study time on those objectives for which you wish to strengthen your knowledge.

* For the Communication and Literacy Skills and General Curriculum tests, candidates may take one or both subtests during the four-hour session.

This practice test may be used as one indicator of potential strengths and weaknesses in your knowledge of the content on the official test. However, because of potential differences in format and difficulty between the practice test and an official MTEL General Curriculum (03) test, it is not possible to predict precisely how you might score on an official MTEL General Curriculum (03) test. Keep in mind that the subareas for which the test weighting is greatest will receive emphasis on this test. Refer to the Test Information Booklet for additional information about how to prepare for the test.

GENERAL CURRICULUM (03) PRACTICE TEST

MATHEMATICS SUBTEST

GENERAL TEST DIRECTIONS

This practice test consists of two subtests: multi-subject (booklet 1) and mathematics (booklet 2). Each multiple-choice question on the practice test has four answer choices. Read each question carefully and choose the ONE best answer. Record each answer on the answer sheet provided.

<u>Sample Question</u>:   1.   What is the capital of Massachusetts?

      A.   Worcester
      B.   New Bedford
      C.   Boston
      D.   Springfield

The correct answer to this question is C. You would indicate that on the answer sheet.

The open-response item assignment on this practice test requires a written response. Directions for the open-response item assignment appear immediately before the assignment.

You may work on the multiple-choice questions and open-response item assignment in any order that you choose. You may wish to monitor how long it takes you to complete the practice test. When taking the actual MTEL General Curriculum (03) test, you will have one four-hour test session in which to complete the test.

Please note that graphic representations included on the test may not be drawn to scale.

MULTIPLE-CHOICE ANSWER SHEET

Mathematics Subtest

Question Number	Your Response		Question Number	Your Response
1			26	
2			27	
3			28	
4			29	
5			30	
6			31	
7			32	
8			33	
9			34	
10			35	
11			36	
12			37	
13			38	
14			39	
15			40	
16			41	
17			42	
18			43	
19			44	
20			45	
21				
22				
23				
24				
25				

MULTIPLE-CHOICE QUESTIONS

1. In the number 2010, the value represented by the digit 1 is what fraction of the value represented by the digit 2?

    A. $\frac{1}{2000}$

    B. $\frac{1}{200}$

    C. $\frac{1}{20}$

    D. $\frac{1}{2}$

2. If P is a positive integer, which of the following must also be a positive integer?

    A. $1 - P$

    B. $\frac{1}{P}$

    C. $\sqrt{P}$

    D. P^2

3. According to an article in a financial journal, a certain company
    earned 3.85 million dollars last year. Based on this report
    of the company's yearly earnings, a person reading the
    article estimates that the company earned an average of
    approximately 30 thousand dollars per month. Which of the
    following statements best describes the reasonableness of
    this estimate for the company's average monthly earnings?

    A.   The estimate is too low by a factor of 100.

    B.   The estimate is too low by a factor of 10.

    C.   The estimate is too high by a factor of 10.

    D.   The estimate is too high by a factor of 100.

4. The mean distance from the earth to the sun is approximately
    93 million miles, or one astronomical unit (AU). The mean
    distance from Neptune to the sun is approximately
    2.794×10^9 miles. What is the approximate mean distance
    from Neptune to the sun in astronomical units?

    A.   30 AU

    B.   300 AU

    C.   3,000 AU

    D.   30,000 AU

5. **Use the expression below to answer the question that follows.**

$$\frac{(32{,}629)(484)}{306{,}751}$$

Which of the following is the best estimate of the value of the expression above?

A.   40

B.   50

C.   400

D.   500

6. **Use the diagram below to answer the question that follows.**
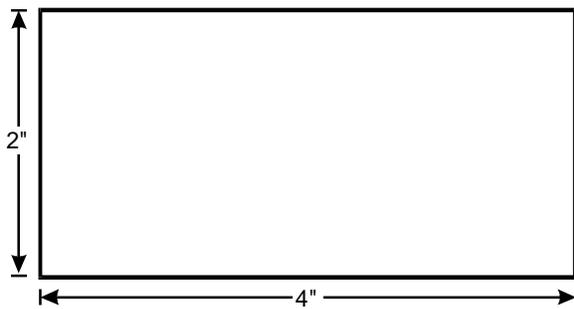
The measurements in the diagram above are shown rounded to the nearest whole number. Which of the following is a possible value of A, the area of the rectangle?

A.   5.0 square inches

B.   5.5 square inches

C.   11.5 square inches

D.   12.0 square inches

7.   **Use the table below to answer the question that follows.**

Store	Discount from Manufacturer's Recommended Price
1	$4 off the price of each game
2	30% discount on each game
3	$\frac{1}{3}$ off the price of two games
4	buy one game and get the second at half price

Samantha wants to buy two computer games, each of which has a manufacturer's recommended price of $20. She checks four different stores and finds the prices of the games discounted as shown in the table above. At which store will Samantha be able to buy the games for the least amount of money?

A.   store 1

B.   store 2

C.   store 3

D.   store 4

8. **Use the procedure below to answer the question that follows.**

$$n = 0.636363...$$
$$100n = 63.636363...$$

$$100n = 63.636363...$$
$$\underline{- \quad n = - \; 0.636363...}$$
$$99n = 63$$

The procedure above shows how to convert a repeating decimal to a fraction. If 0.12561256... is a decimal with four repeating digits, which of the following represents this decimal as a fraction?

A. $\dfrac{1{,}256}{99}$

B. $\dfrac{1{,}256}{999}$

C. $\dfrac{1{,}256}{9{,}999}$

D. $\dfrac{1{,}256}{99{,}999}$

9. **Use the problem below to answer the question that follows.**

> Given that 100 milliliters is equal to approximately 0.4 cup, 205 milliliters is equal to approximately how many cups?

Which of the following expressions models the solution to the problem above?

A. $(100 - 0.4)(205)$

B. 105% of 0.4

C. $(205 - 100)(0.4)$

D. 205% of 0.4

10. **Use the number line below to answer the question that follows.**
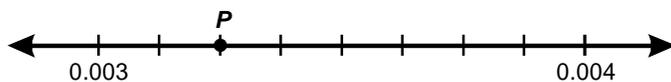
What number is represented by point P on the number line above?

A. 0.0032

B. 0.00325

C. 0.0034

D. 0.00345

11. A book distributor is trying to divide an order of textbooks into equally sized groups for shipping in cartons. The textbooks can be divided into groups of 12, groups of 15, or groups of 18, with no books left over. Which of the following inequalities is satisfied if N is the smallest possible total number of textbooks?

A. $100 \leq N < 150$

B. $150 \leq N < 200$

C. $200 \leq N < 250$

D. $250 \leq N < 300$

12.   The prime factorization of a natural number n can be written as
       $n = pr^2$ where p and r are distinct prime numbers. How many
       factors does n have, including 1 and itself?

       A.   3

       B.   4

       C.   5

       D.   6

13.   Given $pn = 150$ where p is a prime number and n is a natural
       number, which of the following must be true?

       A. p is a factor of either 10 or 15.

       B.   10 is a factor of n.

       C. n is a factor of either 10 or 15.

       D.   15 is a factor of n.

14. The greatest common factor of *n* and 540 is 36. Which of the following could be the prime factorization of *n*?

    A. $2 \cdot 3^2$

    B. $2^2 \cdot 3^3$

    C. $2^4 \cdot 3^2 \cdot 7$

    D. $2^4 \cdot 3^5 \cdot 5$

15. A shipping container measures 8 feet by 12 feet by 24 feet. The container is to be filled with identical cube-shaped boxes, each having sides measuring a whole number of feet. Which of the following expressions represents the smallest number of such identical boxes that could be packed into the container with no empty space remaining?

    A. $\frac{8}{4} + \frac{12}{4} + \frac{24}{4}$

    B. $\frac{8}{4} \cdot \frac{12}{4} \cdot \frac{24}{4}$

    C. $8 \cdot 12 \cdot 24$

    D. $8 + 12 + 24$

16.   **Use the diagram below to answer the question that follows.**
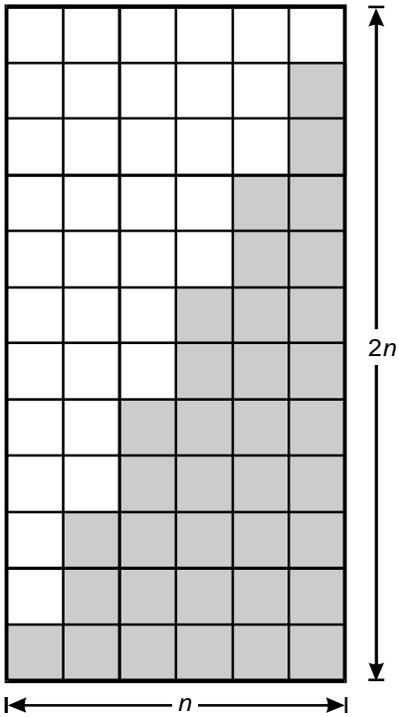
The diagram above could best be used to derive a formula for which of the following quantities?

A.   the sum of the first n consecutive odd integers

B.   the product of the first n consecutive even integers

C.   the sum of the first n consecutive even integers

D.   the product of the first n consecutive odd integers

17. **Use the diagram below to answer the question that follows.**
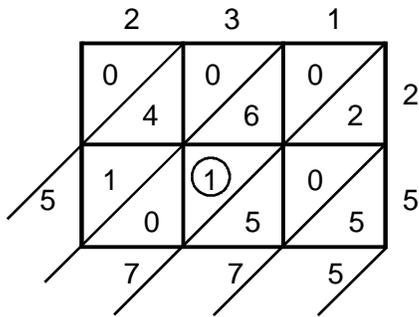
The diagram above demonstrates how the lattice multiplication algorithm is used to multiply 231 by 25 to get the product 5775. What value does the circled digit represent?

A.   1

B.   10

C.   100

D.   1000

18. **Use the samples of a student's work below to answer the question that follows.**

$$\frac{9}{16} \div \frac{3}{4} = \frac{9 \div 3}{16 \div 4} = \frac{3}{4}$$

$$\frac{15}{8} \div \frac{5}{4} = \frac{15 \div 5}{8 \div 4} = \frac{3}{2}$$

$$\frac{5}{12} \div \frac{5}{3} = \frac{5 \div 5}{12 \div 3} = \frac{1}{4}$$

Which of the following statements best describes the mathematical validity of the algorithm that the student appears to be using?

A. It is not valid for any rational numbers.

B. It is valid only when all numerators and denominators are integers.

C. It is valid only when all numerators and denominators are positive integers.

D. It is valid for all rational numbers.

19. **Use the inequality below to answer the question that follows.**

$3 \div x > 4 \div x$

Which of the following inequalities describes all possible values of x?

A. $x < -1$

B. $x < 0$

C. $-1 < x < 0$

D. $0 < x < 1$

20. The expression $(5^{-8} \cdot 7^{-9})$ is equal to which of the following?

    A. $\dfrac{1}{5(35)^8}$

    B. $\dfrac{1}{7(35)^8}$

    C. $\dfrac{5}{(35)^8}$

    D. $\dfrac{7}{(35)^8}$

21. **Use the diagram below to answer the question that follows.**

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40

Which of the following algebraic equations could best be used to explain why, for any three-by-three cross like the one shown above, the sum of the numbers in the vertical rectangle is equal to the sum of the numbers in the horizontal rectangle?

    A. $9x + 14x + 19x = 13x + 14x + 15x$

    B. $5[x + (x + 1) + (x + 2)] = 5x + (5x + 5) + (5x + 10)$

    C. $x + 9 + 14 + 19 = x + 13 + 14 + 15$

    D. $(x - 1) + x + (x + 1) = (x - 5) + x + (x + 5)$

22. **Use the problem below to answer the question that follows.**

    > A landscaper bought some decorative cement blocks from a landscaping supplier. The supplier charged 5% sales tax and the total came to $315. Without the tax, the landscaper could have bought 6 more blocks for the same total cost. How many blocks did the landscaper buy?

    If p represents the price of one block, in which of the following equations does x represent the answer to the problem above?

    A. $0.95px = p(x + 6)$

    B. $1.05p(x + 6) = 315$

    C. $1.05px = p(x + 6)$

    D. $0.95p(x + 6) = 315$

23. A store that sells handcrafted items takes $3.00 per item plus 40% of the sale price for each item sold. The rest of the money from item sales goes to the craftsperson. All items sold cost $5.00 or more. If p represents the sale price of one item, which of the following expressions represents the amount of money the craftsperson gets for each item sold?

    A. $\frac{2}{5}p + 3$

    B. $\frac{2}{5}p - 3$

    C. $\frac{3}{5}p + 3$

    D. $\frac{3}{5}p - 3$

24. **Use the solution procedure below to answer the question that follows.**

$-3x + 25 = 4$
$-3x + 25 - 25 = 4 - 25 = -21 \div (-3) = 7$
$x = 7$

Which of the following is a major flaw in the procedure shown above?

A.   The concept of the opposite of a number is confused with subtraction.

B.   The equal sign is used to connect expressions that are not equal.

C.   The solution contains an error in the arithmetic of signed numbers.

D.   The order of operations between subtraction and division is reversed.

25. **Use the diagram below to answer the question that follows.**
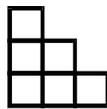
Figure 1   **Figure 2**   **Figure 3**   **Figure 4**
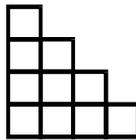
If the pattern continues, how many more small squares are in figure 100 than are in figure 99?

A.   98

B.   99

C.   100

D.   101

26.   **Use the table below to answer the question that follows.**

y					
4	20	28	36	44	52
3	15	23	31	39	47
2	10	18	26	34	42
1	5	13	21	29	37
0	0	8	16	24	32
	0	**1**	**2**	**3**	**4**

x

Each number in the table above represents a value of w that is determined by the values of integers x and y. For example, when $x = 2$ and $y = 1$, $w = 21$. If the pattern continues, what is the value of w when $x = 20$ and $y = 8$?

A.   164

B.   200

C.   208

D.   820

27.   The function r(x) gives the remainder when a whole number x is divided by 10. Which of the following graphs represents r(x)?

A.
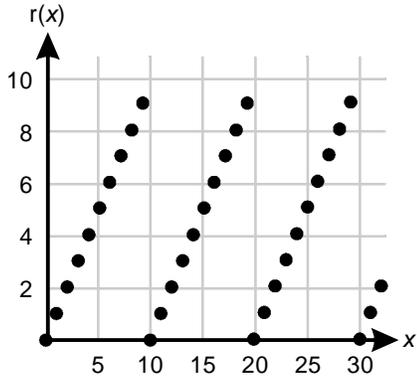
B.
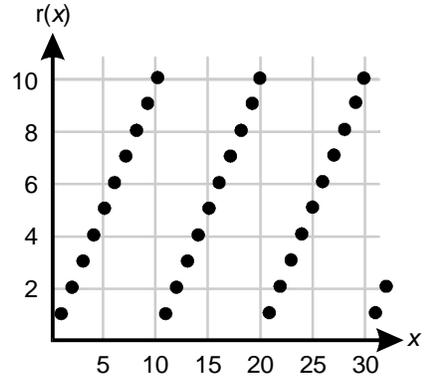
C.
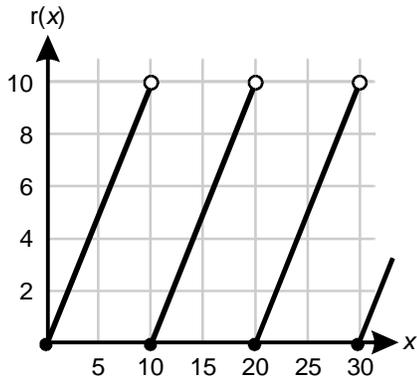
D.
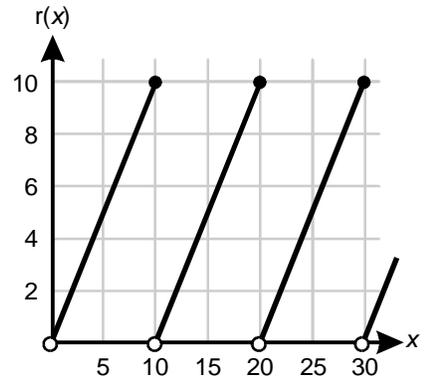
28.   Four pumps begin draining a 5400-gallon pool. At the same time, two pumps begin draining a 4000-gallon pool. Assuming that all of the pumps drain at the same rate, how many gallons are left in the smaller pool when the larger pool is finished being drained?

   A.   1300 gallons

   B.   1350 gallons

   C.   2700 gallons

   D.   2750 gallons

29.   **Use the graph below to answer the question that follows.**
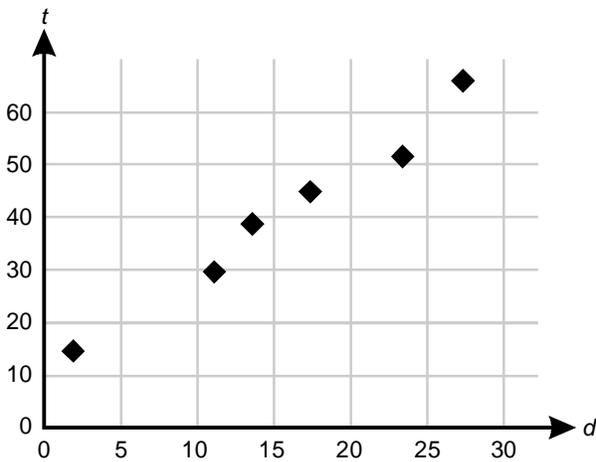
The graph above shows the distance d in miles and the time t in minutes for six bus routes around a city. Which of the following equations best models the relationship between d and t for these bus routes?

   A. $t = d$

   B. $t = d + 10$

   C. $t = 2d$

   D. $t = 2d + 10$

30. **Use the graph below to answer the question that follows.**
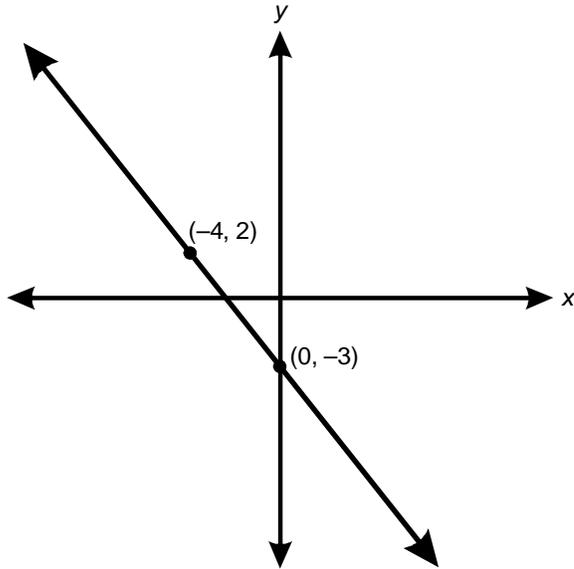
The graph above represents the equation $Wx + 4y = -12$.
What is the value of W?

A. -6

B. -3

C. 3

D. 5

31. **Use the problem below to answer the question that follows.**

A red car and a blue car compete in two 100-mile races. In the first race, both cars leave the starting line at the same time. When the red car crosses the finish line, the blue car has 10 miles left to go. In the second race, both cars start at the same time, but while the blue car begins at the starting line, the red car begins 10 miles behind the starting line.

Assuming that each car's average speed does not change, how far has the blue car traveled in the second race when the red car reaches the finish line?

Which of the following proportions could be used to solve the problem above?

A. $\dfrac{100}{90} = \dfrac{110}{x}$

B. $\dfrac{100}{10} = \dfrac{110}{x}$

C. $\dfrac{100}{90} = \dfrac{90}{x}$

D. $\dfrac{100}{10} = \dfrac{90}{x}$

32. **Use the diagram below to answer the question that follows.**
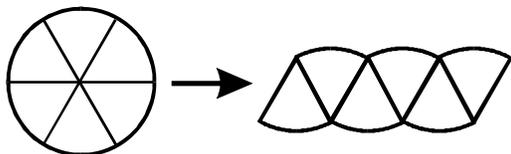
The diagram above is used to describe the relationship between the circumference *c*, the radius *r*, and the area *A* of a circle. Assuming that the circle is divided into enough sections so that the figure on the right approximates a rectangle, which of the following relationships is demonstrated?

A. $A = \frac{1}{2}cr$

B. $A = cr$

C. $A = \frac{3}{2}cr$

D. $A = 2cr$

33. A pretzel company sells pretzels in a cylindrical container with a radius of 10 cm and a height of 30 cm. The company's packaging designers are considering switching to a new cylindrical container with a radius of 20 cm and a height of 15 cm. How does the volume of the proposed new container compare to the volume of the old container?

A. The volume of the new container is 125 cm^3 less than the volume of the old container.

B. The volume of the new container is 5 cm^3 less than the volume of the old container.

C. The volume of the new container is equal to the volume of the old container.

D. The volume of the new container is twice the volume of the old container.

34. A fuel tank was approximately $\frac{1}{8}$ full. After adding $50 worth of fuel, the tank was approximately $\frac{3}{4}$ full. If the fuel costs p dollars per gallon, approximately how many gallons does the tank hold when full?

   A. $\frac{80}{p}$ gallons

   B. $\frac{50}{p}$ gallons

   C.   50p gallons

   D.   80p gallons

35. A homeowner is planning to use carpet tiles to cover the floor of a room measuring 9 feet by 10 feet 8 inches. If the carpet tiles are 8 inches wide and 1 foot long and there are no gaps between the tiles as they are placed on the floor, how many carpet tiles will the homeowner need to cover the floor of the room?

   A.   100

   B.   135

   C.   144

   D.   150

36. **Use the diagram below to answer the question that follows.**
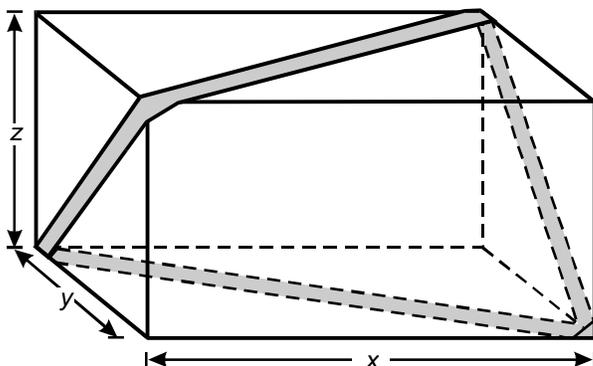
A gift box has dimensions x by y by z. A decorative ribbon is wrapped across the diagonals of the box as shown above. Which of the following expressions represents the approximate total length of the ribbon?

A. $2(\sqrt{xy} + \sqrt{yz})$

B. $2(\sqrt{xy} + \sqrt{yz} + \sqrt{zx})$

C. $2(\sqrt{x^2 + y^2} + \sqrt{y^2 + z^2})$

D. $2(\sqrt{x^2 + y^2} + \sqrt{y^2 + z^2} + \sqrt{z^2 + x^2})$

37. **Use the graph below to answer the question that follows.**
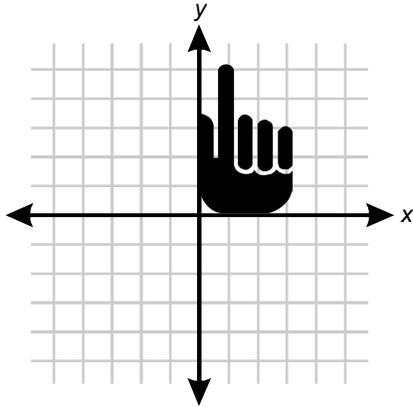
If the image of a pointing hand in the graph above is rotated 180° about the origin and then reflected across the *x*-axis, which of the following graphs will result?

A.
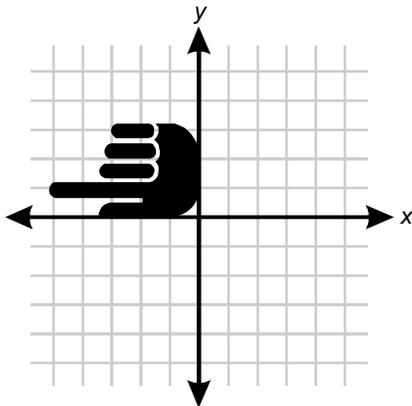
B.
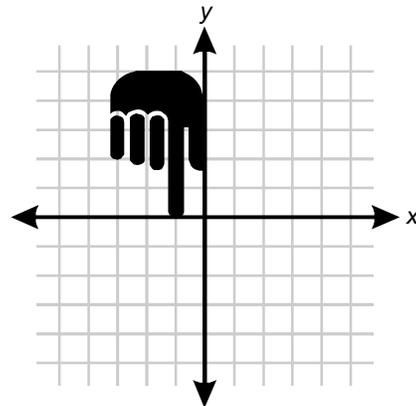
C.
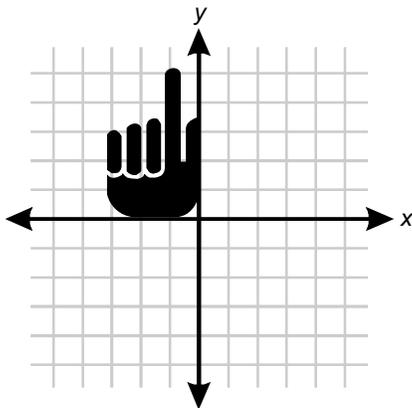
D.
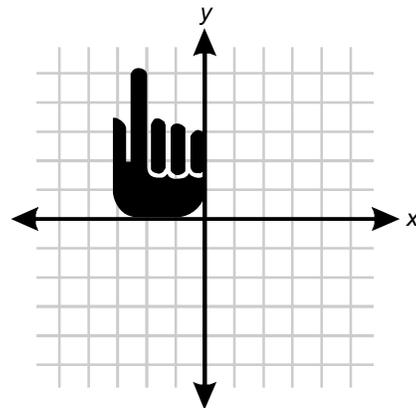
38.   Which of the following nets can be folded to form a square
       pyramid?

A.
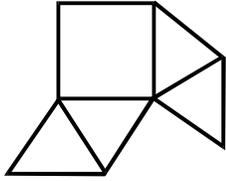
B.
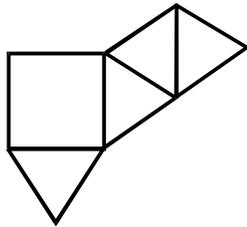
C.
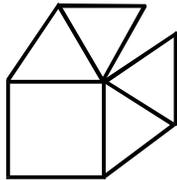
D.
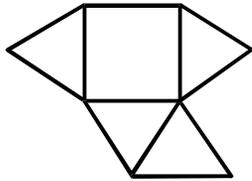
39. **Use the figure below to answer the question that follows.**
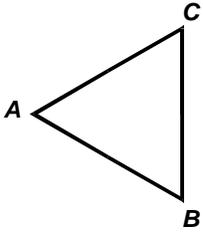
If equilateral triangle *ABC* above represents one of two congruent halves of a figure that has *AB* as a line of symmetry, then the entire figure is a:

A. triangle.

B. rectangle.

C. prism.

D. rhombus.

40. **Use the diagram below to answer the question that follows.**
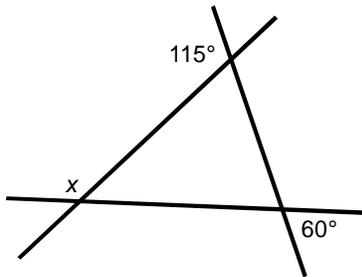
Three straight lines intersect to form a triangle, as shown above. What is the measure of angle *x*?

A. 115°

B. 120°

C. 125°

D. 130°

41. **Use the histogram below to answer the question that follows.**

Sum of Two Number Cubes

Two 6-sided number cubes are rolled simultaneously 10 times. The sums are recorded in the histogram shown above. Which of the following statements can be inferred from the histogram?

A. The mean is less than the median by $\frac{1}{10}$.

B. The mean is greater than the median by $\frac{1}{10}$.

C. The mean is less than the median by $\frac{1}{2}$.

D. The mean is greater than the median by $\frac{1}{2}$.

42. **Use the graph below to answer the question that follows**.

Distribution of Test Scores
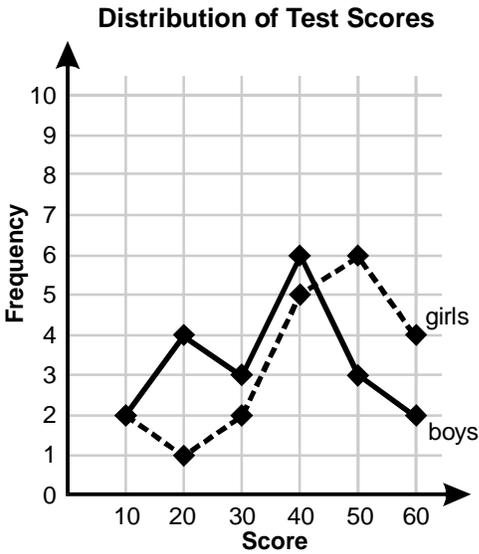
The graph above shows the distribution of scores on a test with possible scores of 10, 20, 30, 40, 50, and 60. The minimum passing score was 40. 20 girls and 20 boys took the test. The percentage of girls passing the test was how much greater than the percentage of boys passing the test?

A.   25%

B.   20%

C.   15%

D.   10%

43.   **Use the table below to answer the question that follows.**

	Milk Shake Flavor Preference		
Age Range	Vanilla	Chocolate	Strawberry
10–19	7	30	15
20–29	10	16	14
30–39	8	14	12
40–49	9	17	9

A marketing company conducted a survey to determine milk shake flavor preferences among five different age groups. Based on the data in the table, what is the probability that a randomly chosen 35-year-old customer will prefer a strawberry-flavored milk shake?

A. $\frac{1}{7}$

B. $\frac{1}{6}$

C. $\frac{6}{25}$

D. $\frac{6}{17}$

44. A child has a set of blocks, of which 4 are square, 5 are round, and 6 are triangular. The child randomly picks a block from the set and gives it to his sister. The child then randomly picks one more block. What is the probability that the first block was round and the second block was triangular?

    A. $\frac{1}{9}$

    B. $\frac{2}{15}$

    C. $\frac{1}{7}$

    D. $\frac{11}{15}$

45. **Use the spinner below to answer the question that follows.**
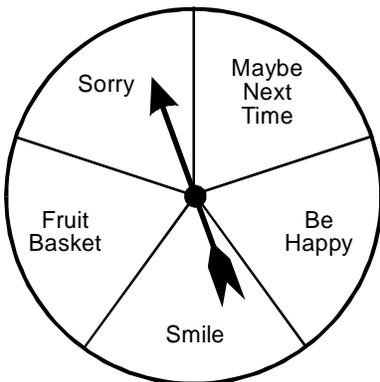
    The host of a party tells her guests that every time the spinner above lands on the section labeled "Fruit Basket," a guest will win a large basket of fruit. If the 180 guests at the party each spin the spinner once, what is the best estimate of the number of fruit baskets that the host will be giving away?

    A. 7

    B. 14

    C. 36

    D. 72

DIRECTIONS FOR THE OPEN-RESPONSE ITEM ASSIGNMENT

This section of the test consists of an open-response item assignment that appears on the following page. You will be asked to prepare a written response of approximately 1–2 pages for the assignment. You should use your time to plan, write, review, and edit your response for the assignment.

Read the topic and directions for the assignment carefully before you begin to work. Think about how you will organize your response.

As a whole, your response to the assignment must demonstrate an understanding of the knowledge of the field. In your response to the assignment, you are expected to demonstrate the depth of your understanding of the subject area by applying your knowledge rather than by merely reciting factual information.

Your response to the assignment will be evaluated based on the following criteria.

- **PURPOSE:** the extent to which the response achieves the purpose of the assignment
- **SUBJECT KNOWLEDGE:** appropriateness and accuracy in the application of subject knowledge
- **SUPPORT:** quality and relevance of supporting evidence
- **RATIONALE:** soundness of argument and degree of understanding of the subject area

The open-response item assignment is intended to assess subject knowledge. Your response must be communicated clearly enough to permit valid judgment of the evaluation criteria by scorers. Your response should be written for an audience of educators in this field. The final version of your response should conform to the conventions of edited American English. Your response should be your original work, written in your own words, and not copied or paraphrased from some other work.

Be sure to write about the assigned topic. Please write legibly. You may not use any reference materials during the test. Remember to review your work and make any changes you think will improve your response.

Write or print your response in the space provided following the assignment.

OPEN-RESPONSE ITEM ASSIGNMENT

Use the information below to complete the exercise that follows.

Students are asked to solve the following problem.

Approximately what fraction of the entire figure shown above is enclosed by the semicircle? (use $\pi = 3.14$)

Student response:

rectangle = 6 × 3 = 18

triangle = $\frac{1}{2}$ × 6 × 4 = 12

semicircle = $\frac{1}{2}$ × 2 × π × radius = 3π

3.14
× 3
9.3.12

18 + 12 + 9.3.12 = 123.12

answer = $\frac{9.3.12}{123.12}$ or about $\frac{90}{120}$ = $\frac{3}{4}$

Use your knowledge of mathematics to create a response in which you analyze the student's work and provide an alternative solution to the problem. In your response, you should do the following:

- Correct any errors or misconceptions evident in the student's work.

- Explain why the response is not mathematically sound. Be sure to
    - provide a correct solution,
    - show your work, and
    - explain your reasoning.

- Solve the problem using an alternative method that could enhance the student's conceptual understanding of ratios and decimal multiplication in the context of the problem.

OPEN-RESPONSE SHEET

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PRACTICE TEST RESULTS

PRACTICE TEST RESULTS OVERVIEW

The practice test provides valuable information regarding your preparedness for the MTEL General Curriculum (03): Mathematics subtest. In this section, you will find information and tools to help you determine your preparedness on the various sections of the test.

Multiple-Choice Questions

A Multiple-Choice Question Answer Key Worksheet is provided to assist you in evaluating your multiple-choice responses. The worksheet contains five columns. The first column indicates the multiple-choice question number, the second column indicates the objective to which the test question was written, and the third column indicates the correct response. The remaining columns are for your use in calculating the number of multiple-choice questions you answered correctly or incorrectly.

An Evaluation Chart for the multiple-choice questions is also provided to help you assess which content covered by the test objectives may require additional study.

Multiple-Choice Question Analyses are provided to explain the correct answer for each question as well as why the other responses are not correct.

Open-Response Item

Evaluation Information, Sample Responses and Analyses, as well as a Scoring Rubric are provided for this item. You may wish to refer to this information when evaluating your practice test response.

Total Test

Practice Test Score Calculation information is provided to help you estimate your score on the practice test. Although you cannot use this practice test to precisely predict how you might score on an official MTEL General Curriculum: Mathematics subtest, you may be able to determine your degree of readiness to take an MTEL test at an operational administration. No passing score has been determined for the practice test.

MULTIPLE-CHOICE QUESTION
ANSWER KEY WORKSHEET

Question Number	Objective Number	Correct Response	Your Response	
			Correct?	**Incorrect?**
1	0016	B		
2	0016	D		
3	0016	B		
4	0016	A		
5	0016	B		
6	0017	B		
7	0017	C		
8	0017	C		
9	0017	D		
10	0017	B		
11	0018	B		
12	0018	D		
13	0018	A		
14	0018	C		
15	0018	B		
16	0019	A		
17	0019	C		
18	0019	D		
19	0019	B		
20	0019	B		
21	0020	D		
22	0020	C		
23	0020	D		
24	0020	B		
25	0021	C		

MULTIPLE-CHOICE QUESTION
ANSWER KEY WORKSHEET (continued)

Question Number	Objective Number	Correct Response	Your Response	
			Correct?	**Incorrect?**
26	0021	B		
27	0021	A		
28	0022	A		
29	0022	D		
30	0022	D		
31	0022	A		
32	0023	A		
33	0023	D		
34	0023	A		
35	0023	C		
36	0024	C		
37	0024	C		
38	0024	B		
39	0024	D		
40	0024	C		
41	0025	A		
42	0025	B		
43	0026	D		
44	0026	C		
45	0026	C		

Count the number of multiple-choice questions you answered correctly:

_____ of 45 multiple-choice questions

MULTIPLE-CHOICE QUESTION
PRACTICE TEST EVALUATION CHART

In the evaluation chart that follows, the multiple-choice questions are arranged in numerical order and by test objective. Check your responses against the correct responses provided to determine how many questions within each objective you answered correctly.

Subarea V: Numbers and Operations

> **Objective 0016: Understand the number system and the concept of place value.**
>
> 1B_____   2D_____   3B_____   4A_____   5B_____   _____/5

> **Objective 0017: Understand integers, fractions, decimals, percents, and mixed numbers.**
>
> 6B_____   7C_____   8C_____   9D_____   10B_____   _____/5

> **Objective 0018: Understand and apply principles of number theory.**
>
> 11B_____   12D_____   13A_____   14C_____   15B_____   _____/5

> **Objective 0019: Understand operations on numbers.**
>
> 16A_____   17C_____   18D_____   19B_____   20B_____   _____/5

Subarea V (Objectives 0016–0019) Total _____/20

MULTIPLE-CHOICE QUESTION
PRACTICE TEST EVALUATION CHART (continued)

Subarea VI: Functions and Algebra

Objective 0020: Understand algebra as generalized arithmetic.	
21D_____ 22C_____ 23D_____ 24B_____	**_____/4**

Objective 0021: Understand the concept of function.	
25C_____ 26B_____ 27A_____	**_____/3**

Objective 0022: Understand linear functions and linear equations.	
28A_____ 29D_____ 30D_____ 31A_____	**_____/4**

Subarea VI (Objectives 0020–0022) Total _____/11

Subarea VII: Geometry and Measurement

Objective 0023: Understand and apply concepts of measurement.	
32A_____ 33D_____ 34A_____ 35C_____	**_____/4**

Objective 0024: Understand and apply concepts of geometry.	
36C_____ 37C_____ 38B_____ 39D_____ 40C_____	**_____/5**

Subarea VII (Objectives 0023–0024) Total _____/9

MULTIPLE-CHOICE QUESTION
PRACTICE TEST EVALUATION CHART (continued)

Subarea VIII: Statistics and Probability

Objective 0025: Understand descriptive statistics.

41A_____   42B_____   **_____/2**

Objective 0026: Understand and apply basic concepts of probability.

43D_____   44C_____   45C_____   **_____/3**

Subarea VIII (Objectives 0025–0026) Total _____/5

OPEN-RESPONSE ITEM EVALUATION INFORMATION

How Open-Response Items Are Scored

Open-response items are scored through a process called focused holistic scoring. Scorers judge the overall effectiveness of the response rather than individual aspects considered in isolation. Scorer judgments are based on the quality of the response, not on length or neatness. Responses must be long enough to cover the topic adequately and scorers must be able to read what is written.

How to Evaluate Your Practice Essay

On the following pages, you will find two "strong" and two "weak" sample responses. PLEASE DO NOT REVIEW THE SAMPLE RESPONSES UNTIL AFTER YOU HAVE WRITTEN YOUR OWN RESPONSE. When you do review the two "strong" and "weak" sample responses and analyses included here, please note the following points:

- ✓ For the purposes of the practice test, responses are identified as "strong" or "weak" rather than given a score point of 1–4.

- ✓ The responses identified as "strong" may contain flaws; however, these responses do demonstrate the performance characteristics of a "strong response."

- ✓ The two "strong" responses demonstrate the examinees' appropriate understanding and application of the subject matter knowledge. However, these responses do not necessarily reflect the full range of "correct answers" that would demonstrate an understanding of the subject matter.

- ✓ The "Analysis" accompanying each "strong" and "weak" response discusses the main attributes of the response, but does not identify all flaws or strengths that may be present.

Compare your practice response to the Sample Responses to determine whether your response is more similar to the strong or weak responses. Also review the Analyses on those pages and the Scoring Rubric to help you better understand the characteristics of strong and weak essays. This evaluation will help you identify specific problems or weaknesses in your practice response. Further information on scoring can be found in the Test Information Booklet and Faculty Guide at www.mtel.nesinc.com and at www.doe.mass.edu/mtel; select "FAQ," then "After the Test."

OPEN-RESPONSE ITEM
SCORING RUBRIC, SAMPLE RESPONSES,
AND ANALYSES

Massachusetts Tests for Educator Licensure®
SCORING RUBRIC FOR SUBJECT TESTS

Performance Characteristics:

Purpose	The extent to which the response achieves the purpose of the assignment.
Subject Matter Knowledge	Accuracy and appropriateness in the application of subject matter knowledge.
Support	Quality and relevance of supporting details.
Rationale	Soundness of argument and degree of understanding of the subject matter.

Scoring Scale:

Score Point	Score Point Description
4	**The "4" response reflects a thorough knowledge and understanding of the subject matter.** • The purpose of the assignment is fully achieved. • There is a substantial, accurate, and appropriate application of subject matter knowledge. • The supporting evidence is sound; there are high-quality, relevant examples. • The response reflects an ably reasoned, comprehensive understanding of the topic.
3	**The "3" response reflects an adequate knowledge and understanding of the subject matter.** • The purpose of the assignment is largely achieved. • There is a generally accurate and appropriate application of subject matter knowledge. • The supporting evidence is adequate; there are some acceptable, relevant examples. • The response reflects an adequately reasoned understanding of the topic.
2	**The "2" response reflects a limited knowledge and understanding of the subject matter.** • The purpose of the assignment is partially achieved. • There is a limited, possibly inaccurate or inappropriate, application of subject matter knowledge. • The supporting evidence is limited; there are few relevant examples. • The response reflects a limited, poorly reasoned understanding of the topic.
1	**The "1" response reflects a weak knowledge and understanding of the subject matter.** • The purpose of the assignment is not achieved. • There is little or no appropriate or accurate application of subject matter knowledge. • The supporting evidence, if present, is weak; there are few or no relevant examples. • The response reflects little or no reasoning about or understanding of the topic.

U	**The response is unrelated to the assigned topic, illegible, primarily in a language other than English, not of sufficient length to score, or merely a repetition of the assignment.**
B	**There is no response to the assignment.**

FIRST SAMPLE WEAK RESPONSE FOR THE
OPEN-RESPONSE ITEM ASSIGNMENT

Using my knowledge of geometric properties, it is hard to see how the student figured out that the rectangle in the given picture is equal to 18 by multiplying 6 x 3. There is no evidence to support where that 6 came from. This student needs to learn how to write out all the steps in the process so that the teacher can see where he is getting the numbers he is using, or it's impossible to tell where he made a mistake. The 6 was probably a guess, and if his guess was right, he correctly did the equations for the rectangle and the triangle.

However, he did the semicircle part wrong. He should have multiplied 3 x 3 x 3.14. This is the width times the height times the radius. Then he would have added 18 for the rectangle, 12 for the triangle, and 28 for the semicircle (rounding up).

18 + 12 + 28 = 58

Then he could have divided 28 by 58 and gotten $\frac{1}{3}$ as his answer. Also, if he guessed the size of the triangle right, he could have seen that the triangle and the semicircle are both the same size. Both are $\frac{1}{3}$ of the entire piece and the triangle takes up just as much space as the semicircle.

ANALYSIS FOR FIRST WEAK RESPONSE TO THE
OPEN-RESPONSE ITEM ASSIGNMENT

This is an example of a weak response because it is characterized by the following:

Purpose: The candidate demonstrates understanding of the general purpose of the problem, but is unable to carry out that assignment. While there is an effort to analyze the student's work, the result is confusion about where the numbers come from and insufficient knowledge to find and correct the student's errors. No alternative problem-solving approach is specified.

Subject Matter Knowledge: The response reveals many weaknesses and gaps in the candidate's subject matter knowledge, such as why the length of the rectangle must be 6, how to get the area of a semicircle, and that, in fact, 28/58 is closer to ½ than ⅓. The given formula for finding the area of a circle, "the width times the height times the radius," reveals the candidate's serious confusion about terms for the properties of a circle. As a result, the candidate can only address the parts of the student's work s/he understands—that the area of the triangle and rectangle are correct, that the area of all three figures need to be added together, and that the proportion of the semicircle would be shown as a fraction.

Support: The candidate's attempts to explain the student's work get bogged down in lack of knowledge, such as, "The 6 was probably a guess." The statements "He should have multiplied $3 \times 3 \times 3.14$. This is the width times the height times the radius" reveal the candidate's confusion about terms for the properties of a circle and what that formula means.   The student's errors in multiplying decimals and in substituting the formula for circumference go unnoticed, so the response is an effort to support ideas that have no significant bearing on the student's work. There is also no step-by-step justification provided for the statements ". . . if he guessed the size of the triangle right, he could have seen that the triangle and the semicircle are both the same size. Both are ⅓ of the entire piece and the triangle takes up just as much space as the semicircle."

Rationale: The response indicates an intuitive sense that the answer must be in the neighborhood of ⅓, but the candidate was unable to use an orderly reasoning process to arrive at that numerical outcome. Two different answers to the semicircle's area are included in this response: one statement, "28 for the semicircle," and one statement that "the triangle and the semicircle are both the same size," which would make the semicircle 12. This difference is not reconciled in the response.

SECOND SAMPLE WEAK RESPONSE FOR THE
OPEN-RESPONSE ITEM ASSIGNMENT

The formula for the area of a circle is πr^2. She used the formula for the circumference of a circle.

$\pi \times 3^2$

$28 \times \dfrac{1}{2} = 14$

$14 + 18 + 12 = 44$

$\dfrac{14}{44} = $ about 30%

A good way to help students see errors in their geometry work is to have them draw or play with actual shapes to see how they fit together. Once they cut shapes into parts, or color parts of shapes, they can put the parts together in different ways to see how alike or different they are in terms of size, like the way a triangle can fit inside a circle.

ANALYSIS FOR SECOND WEAK RESPONSE TO THE
OPEN-RESPONSE ITEM ASSIGNMENT

This is an example of a weak response because it is characterized by the following:

Purpose: This response provides a limited analysis of the student's work, noting only one mathematical element that was incorrect. Consequently, the task of determining how to enhance the student's "conceptual understanding of ratios and decimal multiplication" has not been addressed. The purpose of the assignment has not been achieved.

Subject Matter Knowledge: The single correction of the formula for the area of a circle would not be enough to help the student fully understand the problem and solve it accurately. Without the subject matter knowledge to analyze where the student does and does not possess the requisite understanding, the candidate cannot develop an alternative approach that would help this student. For example, this student's work clearly shows (1) a need for instruction in decimal multiplication (Where does the decimal point in the answer go after you multiply one or more numbers with a decimal?) and (2) the need for practice with how to look at such problems logically as well as mathematically (Why is 3×3.14 going to be closer to 9 than 90? Does "¾" seem close to what you see visually in this diagram?). Instead, the alternative approach given in this response is generalized to "a good way to help students" and does not address *this specific student's* instructional needs.

Support: The response provides no discussion of the mathematical process the candidate used to arrive at the answer of 30%. The description of the student's error in the formula for the area of a circle, while correct, is an incomplete analysis of where the student's work shows understanding and misunderstanding of the underlying mathematical concepts, formulas, processes, and skills. Examples and explanations that are relevant to ratios and decimal multiplication are missing.

Rationale: The response reflects little reasoning about or understanding of the mathematics involved in this problem. The range of mathematical content it represents—different formulas for the areas of geometric shapes, extrapolating data, ratios, decimal multiplication—is unidentified and unexplored. The assignment has been addressed as if it requires only a right answer, instead of as an opportunity for the candidate to demonstrate depth and breadth of understanding about the relevant mathematical content.

FIRST SAMPLE STRONG RESPONSE FOR THE
OPEN-RESPONSE ITEM ASSIGNMENT

The student started off right by determining that the width of the rectangle is 6. The radius of the circle is labeled as 3, which means the diameter is 6. On the diagram, the diameter of the circle is the same as the length of the rectangle. His formulas for finding the area of the rectangle (h x w) and the area of the triangle ($\frac{1}{2}$ bh) are right, and he correctly determined the height of the triangle by subtracting the two segments that are labeled 3 from the total segment labeled 10.

However, the student had trouble with the area of the semicircle because he was using the formula for circumference when he wrote 2 x π x radius ($2\pi r$), instead of the formula for area, which is πr^2. His multiplication is wrong, too. Instead of 3.14 x 3 = 93.12 it should be 3.14 x 3 = 9.42. He has the decimal in the wrong place and didn't carry the 1 over to the next column.

area of rectangle = 6 x 3 = 18

area of triangle = $\frac{1}{2}$ x 6 x 4 = 3 x 4 = 12

area of semicircle = $\frac{1}{2}$ πr^2 = $\frac{1}{2}$ π 9 = 4.5π = 14.13

approximate area of whole = 18 + 12 + 14 = 44

approximate fraction of whole enclosed by semicircle = $\frac{14}{44}$, which reduces to $\frac{7}{22}$, which is approximately $\frac{1}{3}$

I think I would review the formulas for finding the area and circumference of a circle with this student, so that he can use them correctly for problems like these. But for the multiplication mistake I would talk with this student about how multiplying decimals is like multiplying fractions and the decimal places represent fractional parts of a whole. I'd show him how to first estimate an answer by thinking of the whole numbers.

Example: 12.5 x 10.3

The answer should be close to 120 because 12 x 10 is 120, while .5 and .3 are fractional parts of a whole--5 tenths and 3 tenths.

Then we could actually multiply $\frac{5}{10}$ x $\frac{3}{10}$ to show that the answer results in hundredths $\left(\frac{15}{100}\right)$, which helps to explain how to figure out how many decimal places are needed in the answer when multiplying two or more decimals.

Then we could go back to the problem and try first estimating the answer to 3.14 x 3 (around 9), then do it by writing the decimal as a fraction $\left(\frac{14}{100}\right)$ and multiplying it times $\frac{3}{1}$, which equals $\frac{42}{100}$.

ANALYSIS FOR FIRST STRONG RESPONSE TO THE OPEN-RESPONSE ITEM ASSIGNMENT

This is an example of a strong response because it is characterized by the following:

Purpose: All aspects of this assignment have been fully addressed. The first two paragraphs of the response provide a thorough analysis of each part of the student's work. That is followed by a correct, step-by-step problem-solving process and result and an alternative problem-solving approach that addresses a specific need the candidate identified to enhance this student's conceptual understanding of the rules for multiplying decimals.

Subject Matter Knowledge: The response is strengthened by the analysis of what the student did correctly as well as a discussion of where errors occurred in the student's work. By reviewing what the student did right, the candidate is demonstrating specific relevant mathematical knowledge: how to determine the length and width of the rectangle, the formulas for finding the area of the rectangle and the triangle, how to determine the height of the triangle, the formulas for finding the perimeter and the area of a circle. The errors noted with decimal multiplication had a significant impact on this student's work. The discussion of the errors and an alternative approach to decimal multiplication further reveal the candidate's substantial and accurate subject matter knowledge. The result is that the response demonstrates comprehensive understanding of every mathematical element of the problem.

Support: The analysis of the student's work provides high-quality support that is often quite detailed, such as ". . . the width of the rectangle is 6. *The radius of the circle is labeled as 3, which means the diameter is 6. On the diagram, the diameter of the circle is the same as the length of the rectangle*." To support the error in multiplication, the candidate supplies the correct answer and notes two of the student's errors in that section of the solution. "*He has the decimal in the wrong place and didn't carry the 1 over to the next column*." The example showing how to relate decimal multiplication to multiplying fractions is directly relevant to enhancing this student's conceptual understanding of that specific mathematical process.

Rationale: The candidate's depth of understanding is made clear by the manner in which this response deals with every aspect of the student's work. No detail is overlooked; the candidate provides reasons why each step in the student's response is either correct or incorrect. The final section of the response, explaining where and why to focus on decimal multiplication with this student, and then showing a concrete example of how to do this, makes the candidate's reasoning process clear.

SECOND SAMPLE STRONG RESPONSE FOR THE
OPEN-RESPONSE ITEM ASSIGNMENT

The process the student followed was correct and included all of the necessary steps. First she got the area of the rectangle and the triangle, then she tried to get the area of the semicircle. She knew that the next steps would be to add together all of those areas and then make a ratio out of how many parts of the total the semicircle was. Her thinking about how to solve the problem shows a good understanding of the process.

Sometimes, however, this student's answers don't make sense. When she got the answer of $\frac{3}{4}$ she should have visually checked that against the diagram to see if it was logical. Just by looking at the diagram you can tell that the answer should be close to $\frac{1}{3}$, and that $\frac{3}{4}$ is much too large. Also when the student multiplied 3.14 x 3 and got an answer of 93.12, she should have been able to see that it didn't make sense to get such a high number from multiplying just a little more than 3 times 3, and that her answer should have been close to 9. She doesn't seem to know how to use estimation or how to multiply decimals. The other mistake she made is that she had the wrong formula for the area of a circle, which is πr^2, not $2\pi r$. The correct solution is

rectangle = 6 x 3 = 18

triangle = $\frac{1}{2}$ x 6 x 4 = 12

semicircle = $\frac{1}{2}$ π 3² = $\frac{1}{2}$ x 3.14 x 9 = 1.57 x 9 = 14.13 (rounded to 14)

$\frac{14}{18 + 12 + 14} = \frac{14}{44} = \frac{7}{22}$ (which is about $\frac{1}{3}$)

It may be that this student was trying to line up the decimal points when she did decimal multiplication, because her decimal points are right under each other.

```
 3.14
x   3
93.12
```

I would show her how to look at those numbers instead as if they are whole numbers and multiply them just the same way, then put the decimal point in the answer based on how many decimal places are in the numbers that are multiplied. For example:

```
  2.111
x   .62
 4222
12666
1.30882  (5 decimal places all together as there are 3 in 2.111 and 2 in .62)
```

ANALYSIS FOR SECOND STRONG RESPONSE TO THE
OPEN-RESPONSE ITEM ASSIGNMENT

This is an example of a strong response because it is characterized by the following:

Purpose: The response demonstrates a full understanding of the purpose of this assignment. The candidate first provides a more holistic analysis of the student's effort, then discusses specific conceptual and computational errors, shows the correct solution, and provides an example of corrective instruction. The result is a thorough response to all parts of the assignment.

Subject Matter Knowledge: This candidate's analysis of the student's work shows a firm grasp of both concepts and procedures. The candidate finds merit in the student's approach to the problem, describing the mathematical knowledge that the student's step-by-step process reveals. However, the student's work *does* show a troubling lack of attention to "reasonableness," and the need to pay attention to that issue is carefully analyzed. Two key computation errors are explained, and the solution given shows accurate and appropriate subject matter knowledge for this assignment. The instructional example for decimal multiplication is directly related to the error this student made when multiplying 3.14×3.

Support: Every paragraph of this response begins with a general statement about an aspect of the student's work, which is followed by specific examples and explanations that support that statement.   Examples of strong support include ". . . add together all of those areas and then *make a ratio out of* how many parts of the total the semicircle was" and "Just by looking at the diagram you can tell that the answer should be *close to ⅓*, and that *¾ is much too large*," and ". . . it didn't make sense to get such a high number from *multiplying just a little more than 3 times 3*."

Rationale: The candidate's reasoning is clear and stays focused on the student's work and what it indicates about the student's mathematical knowledge. The different aspects of the assignment are reasoned through using precise mathematical concepts and vocabulary, such as ratio, decimal multiplication, and whole numbers. The entire response holds together as an integrated whole that demonstrates the depth and breadth of the candidate's understanding of math concepts and skills in the areas of algebra, geometric measurement, and computation that are integral to this problem.

PRACTICE TEST SCORE CALCULATION

The practice test score calculation is provided so that you may better gauge your performance and degree of readiness to take an MTEL test at an operational administration. Although the results of this practice test may be used as one indicator of potential strengths and weaknesses in your knowledge of the content on the official test, it is not possible to predict precisely how you might score on an official MTEL test.

The Sample Responses and Analyses for the open-response items may help you determine whether your responses are more similar to the strong or weak samples. The Scoring Rubric can also assist in estimating a score for your open responses. You may also wish to ask a mentor or teacher to help evaluate your responses to the open-response questions prior to calculating your total estimated score.

How to Calculate Your Practice Test Score

Review the directions in the sample below and then use the blank practice test score calculation worksheet on the following page to calculate your estimated score.

SAMPLE

Multiple-Choice Section

Enter the total number of multiple-choice questions you answered correctly: 31

Use Table 1 below to convert that number to the score and write your score in **Box A**:   **A:**   220

Open-Response Section

Enter the number of points (1 to 4) for your open-response question: 3

Use Table 2 below to convert that number to the score and write your score in **Box B**:   **B:**   24

Total Practice Test Score (Estimated MTEL Score)

Add the numbers in **Boxes A and B** for an estimate of your MTEL score: **A + B =**   244

Practice Test Score Calculation Worksheet: General Curriculcum Mathematics Subtest

Table 1:

Number of Multiple-Choice Questions Correct	Estimated MTEL Score		Number of Multiple-Choice Questions Correct	Estimated MTEL Score
0 to 6	116		25 to 27	197
7 to 9	127		28 to 30	208
10 to 12	139		31 to 33	220
13 to 15	150		34 to 36	231
16 to 18	162		37 to 39	243
19 to 21	174		40 to 42	255
22 to 24	185		43 to 45	266

Table 2:

Number of Open-Response Question Points	Estimated MTEL Score
1	12
2	18
3	24
4	30

Print the form below to calculate your estimated practice test score.

Multiple-Choice Section

Enter the total number of multiple-choice questions you answered correctly:

Use Table 1 above to convert that number to the score and write your score in **Box A**:   **A:**

Open-Response Section

Enter the number of points (1 to 4) for your open-response question:

Use Table 2 above to convert that number to the score and write your score in **Box B**:   **B:**

Total Practice Test Score (Estimated MTEL Score)

Add the numbers in **Boxes A and B** for an estimate of your MTEL score: **A + B =**