Massachusetts
Tests for Educator Licensure® (MTEL®)

MTEL®

Mathematics (09)

PRACTICE TEST

All examinees taking the Mathematics (09) test will be provided with a scientific calculator with functions that include the following: addition, subtraction, multiplication, division, square root, percent, sine, cosine, tangent, exponents, and logarithms. Refer to "Test Selection" in the current MTEL registration bulletin for more information.

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06/10

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INTRODUCTION

This document is a printable version of the Massachusetts Tests for Educator Licensure® (MTEL®) Mathematics (09) Online Practice Test.

This practice test is a sample test consisting of 100 multiple-choice questions and 2 open-response item assignments. An Answer Key Worksheet, Answer Sheet, and Evaluation Chart by test objective are included for the multiple-choice questions. Blank Response Sheets, Evaluation Information, and Sample Responses and Analyses, as well as a Scoring Rubric, are included for the open-response items. Lastly, there is a Practice Test Score Calculation worksheet.

PURPOSE OF THE PRACTICE TEST

The practice test is designed to provide an additional resource to help you effectively prepare for the MTEL Mathematics (09) test. The primary purpose of the practice test is to help you become familiar with the structure and content of the test. It is also intended to help you identify areas in which to focus your studies. Education faculty and administrators of teacher preparation programs may also find this practice test useful as they help students prepare for the official test.

TAKING THE PRACTICE TEST

In order to maximize the benefits of the practice test, it is recommended that you take this test under conditions similar to the conditions under which the official MTEL tests are administered. Try to take the practice test in a quiet atmosphere with few interruptions and limit yourself to the four-hour time period* allotted for the official test administration. You will find your results to be more useful if you refer to the answer key only after you have completed the practice test.

INCORPORATING THE PRACTICE TEST IN YOUR STUDY PLAN

Although the primary means of preparing for the MTEL is your college education, adequate preparation prior to taking or retaking the MTEL test is strongly recommended. How much preparation and study you need depends on how comfortable and knowledgeable you are with the content of the test.

The first step in preparing to take the MTEL is to identify what information the test will address by reviewing the objectives for your field. A complete, up-to-date list of the Test Objectives is included in the Test Information Booklet for each test field. The test objectives are the core of the testing program and a helpful study tool. Before taking or retaking the official test, focus your study time on those objectives for which you wish to strengthen your knowledge.

This practice test may be used as one indicator of potential strengths and weaknesses in your knowledge of the content on the official test. However, because of potential differences in format and difficulty between the practice test and an official MTEL Mathematics (09) test, it is not possible to predict precisely how you might score on an official MTEL Mathematics (09) test. Keep in mind that the subareas for which the test weighting is greatest will receive emphasis on this test. Refer to the Test Information Booklet for additional information about how to prepare for the test.

* For the Communication and Literacy Skills test, candidates may take one or both subtests during the four-hour session.

MATHEMATICS
PRACTICE TEST

Candidates taking the Mathematics test (field 09) will be provided with the formulas shown below at the test administration.

FORMULAS

Formula	Description
$V = \frac{1}{3}Bh$	Volume of a right circular cone and a pyramid
$A = 4\pi r^2$	Surface area of a sphere
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$A = \pi r\sqrt{r^2 + h^2}$	Lateral surface area of a right circular cone
$S_n = \frac{n}{2}[2a + (n-1)d] = n\left(\frac{a + a_n}{2}\right)$	Sum of an arithmetic series
$S_n = \frac{a(1 - r^n)}{1 - r}$	Sum of a geometric series
$\sum_{n=0}^{\infty} ar^n = \frac{a}{1 - r}, \lvert r \rvert < 1$	Sum of an infinite geometric series
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Midpoint formula
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Law of sines
$c^2 = a^2 + b^2 - 2ab\cos C$	Law of cosines
$s^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n - 1}$	Variance
$s = r\theta$	Arc length
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula

GENERAL TEST DIRECTIONS

This practice test consists of two sections: (1) a multiple-choice question section and (2) an open-response item assignment section. Each multiple-choice question on the practice test has four answer choices. Read each question carefully and choose the ONE best answer. Record each answer on the answer sheet provided.

Sample Question: 1. What is the capital of Massachusetts?

A. Worcester
B. New Bedford
C. Boston
D. Springfield

The correct answer to this question is C. You would indicate that on the answer sheet.

The open-response section of this practice test requires written responses. Directions for the open-response item assignments appear immediately before those assignments.

You may work on the multiple-choice questions and open-response item assignments in any order that you choose. You may wish to monitor how long it takes you to complete the practice test. When taking the actual MTEL Mathematics (09) test, you will have one four-hour test session in which to complete the test.

MULTIPLE-CHOICE ANSWER SHEET

Question Number	Your Response
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Question Number	Your Response
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Question Number	Your Response
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MULTIPLE-CHOICE QUESTIONS

1. In the hexadecimal (base 16) number "FE03," which of the following places is occupied by the value E?

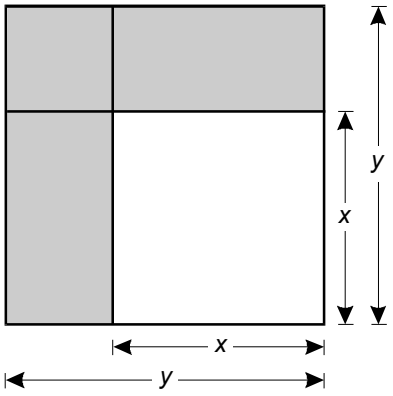
   A. the 4,096s place

   B. the 256s place

   C. the 64s place

   D. the 16s place

2. **Use the diagram below to answer the question that follows.**



   Which of the following identities is best represented by the diagram above?

   A. $2y + 2x = (y + x) + (y + x)$

   B. $2y - 2x = (y - x) + (y - x)$

   C. $y^2 + x^2 = (y + x)^2 - 2xy$

   D. $y^2 - x^2 = (y - x)^2 + 2x(y - x)$

3. A digital image is made up of 960 rows of dots with 1,920 dots in each row. Which of the following would be the most reasonable approach to estimating the total number of dots in the image?

   A. Add 96 to 192, giving 288. Putting back the zero on the end yields 2,880.

   B. Because 960 is just under 1,000 and 1,920 is just over 1,900, the answer should be close to 2,900.

   C. Multiply 96 by 192, giving 18,432. Putting back the two zeros on the end yields 1,843,200.

   D. Because 960 is just under 1,000 and 1,920 is just over 1,900, the answer should be close to 1,900,000.

4. Which of the following characteristics differentiates a positional numeration system from a nonpositional numeration system?

   A. A positional system must have a symbol for zero.

   B. A positional system must use a distinct cipher in each place value.

   C. A nonpositional system does not have a base.

   D. A nonpositional system relies on hieroglyphics instead of abstract symbols.

Use the computation below to answer the two questions that follow.

$$
\begin{array}{r}
cd \\
d\overline{)bae} \\
-be \\
\hline
ce \\
-cd \\
\hline
b
\end{array}
$$

The letters a, b, c, d, and e represent the specific digits of the numbers involved in the long division computation above. The letters a, b, c, d, and e are not variables.

5. Which of the following equations correctly represents this computation?

   A. $bae \div d = cd.b$

   B. $bae \div d = cd + b$

   C. $bae \div d = cd\ \frac{b}{d}$

   D. $bae \div d = cd\ \frac{b}{bae}$

6. Which modular arithmetic statement follows from the computation?

   A. $bae \equiv b \pmod{d}$

   B. $bae \equiv cd \pmod{b}$

   C. $bae \div cd \equiv b \pmod{d}$

   D. $bae \div d \equiv cd \pmod{b}$

Use the theorem and proof below to answer the two questions that follow.

Theorem:
If m^2 is even, then m is even.

Proof:

given:	m^2 is even
step 1:	assume m is odd
step 2:	$m = 2k + 1$ for some integer k
step 3:	$m^2 = (2k + 1)^2$
step 4:	$m^2 = 4k^2 + 4k + 1$
step 5:	$m^2 = 2(2k^2 + 2k) + 1$
step 6:	m^2 is odd
step 7:	the assumption is false
step 8:	m is even

7. Which of the following properties justifies the statement in step 5?

   A. the reflexive property

   B. the distributive property

   C. the associative property

   D. the commutative property

8. This proof is an example of:

   A. an indirect proof.

   B. a conditional proof.

   C. proof by syllogism.

   D. proof by mathematical induction.

9. Which of the following is equivalent to the expression $\left(\frac{x^p}{y^{2q}}\right)^{-2}\left(\frac{x^{3p}}{y^p}\right)^q$?

A. $x^{\frac{3q}{2}}y^{\frac{4}{p}}$

B. $x^{\frac{3q}{2p}}y^{2q}$

C. $x^{3q-2p}y^{4q-2}$

D. $x^{p(3q-2)}y^{q(4-p)}$

10. **Use the sequence below to answer the question that follows.**

The Mandelbrot Sequence

1st term: c
2nd term: $c^2 + c$
3rd term: $(c^2 + c)^2 + c$
4th term: $[(c^2 + c)^2 + c]^2 + c$

Which of the following would be the third term of the Mandelbrot sequence for $c = \frac{1}{2}i$?

A. $\frac{9}{16}$

B. $1\frac{1}{16}i$

C. $-\frac{3}{16} + \frac{1}{4}i$

D. $-\frac{3}{16} + \frac{1}{2}i$

11. Which of the following sets of numbers is closed under subtraction?

    A. the whole numbers

    B. the real numbers

    C. the irrational numbers

    D. the imaginary numbers

12. A number is considered rational if it can be:

    A. located on a number line.

    B. found by solving an algebraic equation.

    C. expressed as the quotient of an integer and a non-zero integer.

    D. represented by a construction with a compass and a straightedge.

13. If the least common multiple of two integers is equal to their product, then which of the following statements must be true?

    A. They have no prime factors in common.

    B. They have some but not all prime factors in common.

    C. They contain exactly the same set of prime factors raised to different powers.

    D. They contain exactly the same set of prime factors raised to the same powers.

14. One of the basic premises behind Euclid's algorithm states that if $a = bt + r$, for integers a, b, t, and r, then:

    A. the greatest common factor of a and b is equal to the greatest common factor of b and t.

    B. the greatest common factor of a and b is equal to the greatest common factor of b and r.

    C. the least common multiple of a and b is equal to the least common multiple of b and t.

    D. the least common multiple of a and b is equal to the least common multiple of b and r.

15. Assume that p, q, r, and s are prime. If $p^3q^2 = r^3s^2$, then, according to the fundamental theorem of arithmetic,:

    A. $q = s$.

    B. $q > r$.

    C. $p^3 = s^2$.

    D. $p^3 < s^2$.

16. a, b, and c are integers such that $0 < a < b < c$. If c is divisible by b and c is divisible by a, then which of the following statements must be true?

    A. b is divisible by a.

    B. c is divisible by $(a \bullet b)$.

    C. $(a \bullet b)$ is divisible by c.

    D. $(a \bullet c)$ is divisible by b.

17. **Use the table below to answer the question that follows.**

	f(2) – f(1)	f(3) – f(2)	f(4) – f(3)	f(5) – f(4)
$f_1(x)$	1	4	9	16
$f_2(x)$	2	2	2	2
$f_3(x)$	3	5	7	9
$f_4(x)$	4	5	7	10

The table above shows the first differences of four functions. Based on their second differences, which of the following functions is quadratic?

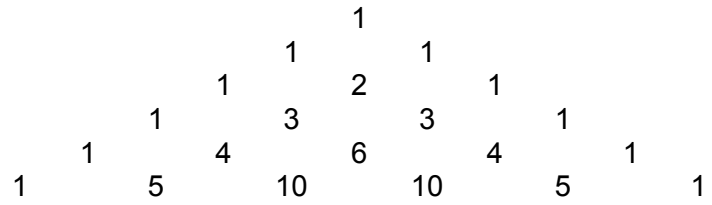
A. $f_1(x)$

B. $f_2(x)$

C. $f_3(x)$

D. $f_4(x)$

18. **Use the portion of Pascal's triangle below to answer the question that follows.**



Based on the portion of Pascal's triangle above, which of the following values is the coefficient of the x^3y term in the expansion of $(x + y)^4$?

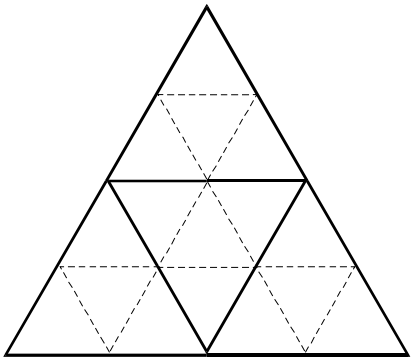
A. 3

B. 4

C. 5

D. 6

19. **Use the diagram below to answer the question that follows.**



Connecting the midpoints of the three sides of an equilateral triangle will give rise to four smaller congruent equilateral triangles. Repeating this process on each of the four smaller triangles will result in a total of 16 yet smaller equilateral triangles, as shown above. Repeating this process *n* times will result in how many of the smallest-size triangles?

A. n^2

B. $4n^2$

C. 2^n

D. 4^n

20. **Use the recursive definition below to answer the question that follows.**

$a_n = a_{n-2} + a_{n-1} + n$
$a_1 = 1$
$a_2 = -2$

Based on the recursive definition above, which of the following is the value of a_6?

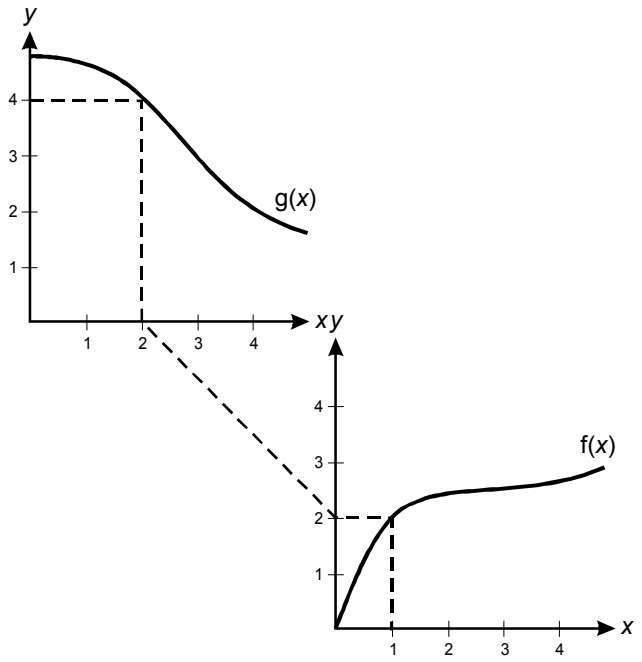
A. -7

B. 13

C. 21

D. 41

21. **Use the graphs below to answer the question that follows.**



Which of the following equations is represented by the graphs above?

A. f(g(1)) = 4

B. f(g(4)) = 1

C. g(f(1)) = 4

D. g(f(4)) = 1

22. For which of the following functions is the domain equal to the range?

A. $f(x) = x^2$

B. $f(x) = \frac{1}{x + 1}$

C. $f(x) = \sqrt{2x}$

D. $f(x) = |\sqrt[3]{x}|$

23. An employer pays \$7 per hour plus an extra \$2 per hour for every hour worked beyond 8 hours up to a maximum daily wage of \$92. Which of the following piecewise-defined functions correctly represents wages, *w*, in dollars, as a function of time, *t*, in hours?

A. $w(t) = \begin{cases} 7t & \text{for } 0 \le t \le 8 \\ 2t & \text{for } 8 < t < 12 \\ 92 & \text{for } t \ge 12 \end{cases}$

B. $w(t) = \begin{cases} 7t & \text{for } 0 \le t \le 8 \\ 9t & \text{for } 8 < t < 10 \\ 92 & \text{for } t \ge 10 \end{cases}$

C. $w(t) = \begin{cases} 7t & \text{for } 0 \le t \le 8 \\ 56 + 2t & \text{for } 8 < t < 10 \\ 92 & \text{for } t \ge 10 \end{cases}$

D. $w(t) = \begin{cases} 7t & \text{for } 0 \le t \le 8 \\ 56 + 9(t - 8) & \text{for } 8 < t < 12 \\ 92 & \text{for } t \ge 12 \end{cases}$

24. **Use the function below to answer the question that follows.**

$$f(x) = \frac{\sqrt{x}}{(x + 2)^3}$$

Which of the following specifies the domain of the function?

A. $(-\infty, -2) \cup (-2, \infty)$

B. $(-2, \infty)$

C. $[0, \infty)$

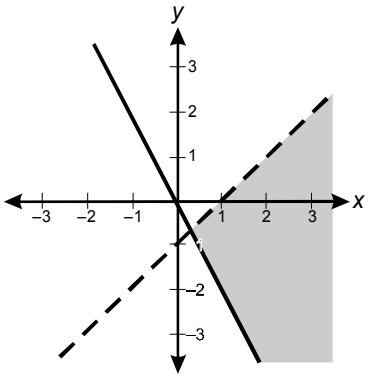
D. $(-\infty, -2) \cup [0, \infty)$

25. **Use the inequalities below to answer the question that follows.**

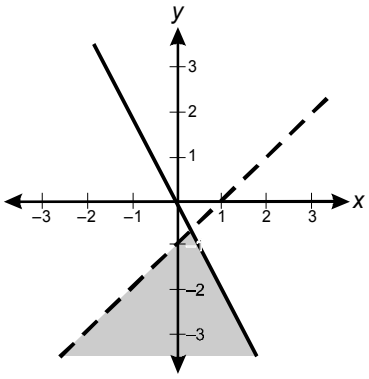
$2x + y \geq 0$
$x - y < 1$

Which of the following graphs shows the solution to the system of linear inequalities?

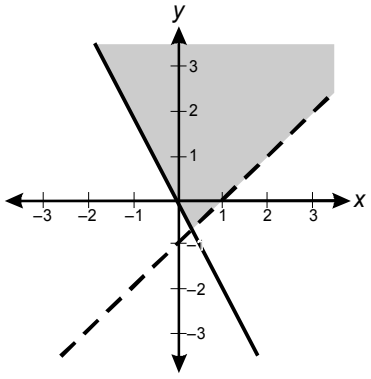
A.



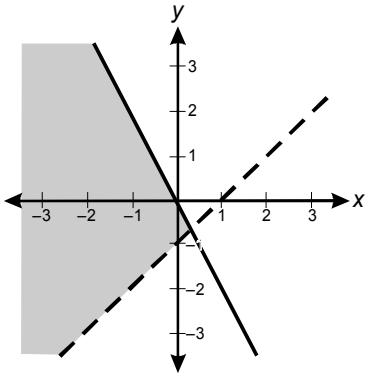
B.



C.



D.



26. A line passes through the points (–3, 5) and (4, 1). Which of the following equations represents the line?

A. $4x - 7y = 23$

B. $4x + 7y = 23$

C. $7x - 4y = 32$

D. $7x + 4y = 32$

27. **Use the system of linear equations below to answer the question that follows.**

$2x + 3y = 4$
$5x - 2y = 6$

Which of the following ordered pairs is the solution to the system above?

A. $\left(\frac{8}{19}, \frac{26}{19}\right)$

B. $\left(\frac{26}{19}, \frac{8}{19}\right)$

C. $\left(\frac{10}{11}, \frac{8}{11}\right)$

D. $\left(\frac{8}{11}, \frac{10}{11}\right)$

28. A line passing through the point (–2, 11) has a *y*-intercept of 5. The line also passes through the point (x, –22). What is the value of x?

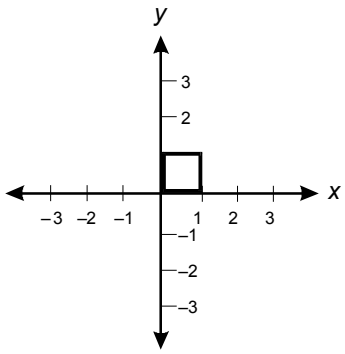
A. –13

B. –9

C. 9

D. 13

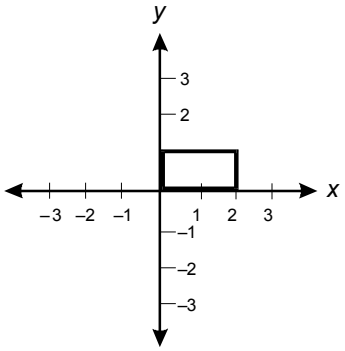
29. **Use the graph and matrix below to answer the question that follows.**



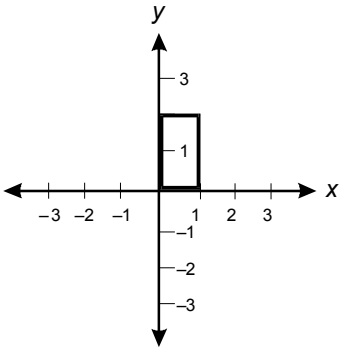
$T = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

Which of the following graphs results from using linear transformation matrix *T* on the graph above?

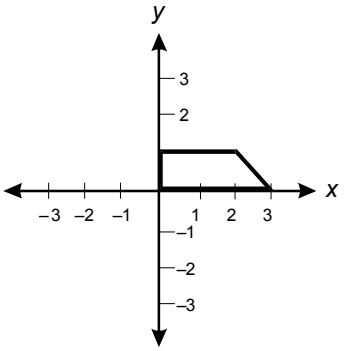
A.



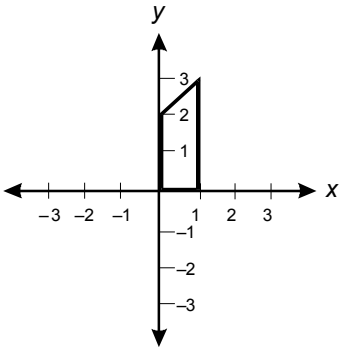
B.



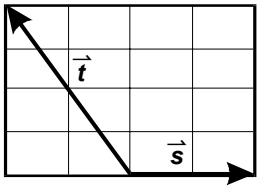
C.



D.

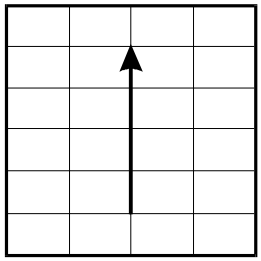


30. **Use the diagram below to answer the question that follows.**

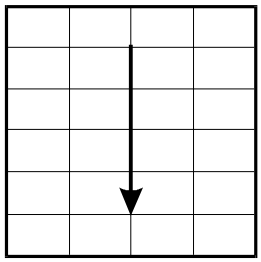


Which of the following vectors represents $\vec{t} + \vec{s}$?

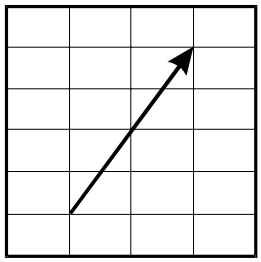
A.



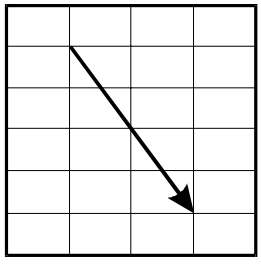
B.



C.



D.



31. **Use the matrix definitions below to answer the question that follows.**

$$Q = \begin{bmatrix} 2 & 4 \\ 3 & 3 \end{bmatrix} \quad R = \begin{bmatrix} 0 & -2 & -1 \\ 7 & 2 & -1 \\ 6 & 5 & 5 \end{bmatrix} \quad S = \begin{bmatrix} 3 & 2 \\ -4 & 9 \\ 4 & 8 \end{bmatrix}$$

Based on the definitions above, which of the following matrix products is defined?

A. QR

B. QS

C. SQ

D. SR

32. **Use the matrices below to answer the question that follows.**

$$A = \begin{array}{r} \\ \text{Mon} \\ \text{Tue} \\ \text{Wed} \\ \text{Thu} \\ \text{Fri} \end{array} \begin{array}{c} \begin{array}{ccc} \text{breakfast} & \text{lunch} & \text{dinner} \end{array} \\ \begin{bmatrix} 10 & 3 & 10 \\ 0 & 10 & 10 \\ 2 & 0 & 5 \\ 6 & 5 & 10 \\ 4 & 8 & 12 \end{bmatrix} \end{array} \qquad B = \begin{array}{r} \\ \text{breakfast} \\ \text{lunch} \\ \text{dinner} \end{array} \begin{array}{c} \text{cost} \\ \begin{bmatrix} \$\ 5 \\ \$10 \\ \$20 \end{bmatrix} \end{array}$$

Matrix A represents the numbers of breakfasts, lunches, and dinners served on each of five consecutive days. Matrix B represents the cost of each breakfast, lunch, and dinner. Which of the following is represented by the product of matrix A and matrix B?

A. the total costs of all meals combined for each day

B. the total costs for each type of meal on each day

C. the total costs for each type of meal for the entire five days

D. the total cost of all meals combined for the entire five days

33. **Use the function below to answer the question that follows.**

$f(x) = 2x^2 - 24x + 6$

Which of the following is the maximum value of $f(x)$ on the interval $[-3, 5]$?

A. 96

B. 0

C. –64

D. –66

34. $(-\infty, -11) \cup (5, \infty)$ is the solution to which of the following inequalities?

A. $x^2 - 6x - 55 < 0$

B. $x^2 - 6x - 55 > 0$

C. $x^2 + 6x - 55 < 0$

D. $x^2 + 6x - 55 > 0$

35. **Use the equation below to answer the question that follows.**

$ax^2 + bx + c = 0$

If a, b, and c are real numbers, which of the following could be the entire solution to the equation above?

A. $x = 3$, or $x = 1 - i$

B. $x = 2i$, or $x = -2i$

C. $x = 1 - i$, or $x = 1 + 2i$

D. $x = 1 + i$

36. A parabola has vertex (2, 1) and a vertical axis of symmetry. The parabola passes through points (3, 3) and (5, p). What is the value of p?

A. 4

B. 7

C. 18

D. 19

37. Which of the following is equivalent to the expression $\frac{x-3}{x+2}+\frac{x+2}{x-3}$?

A. $\frac{2x^2+13}{x^2-x-6}$

B. $\frac{x^2-6x+9}{x^2+4x-6}$

C. $\frac{2x^2-2x+13}{x^2-x-6}$

D. $\frac{2x^2-2x+13}{2x^2-2x+3}$

38. The light intensity, I, of a light bulb varies inversely as the square of the distance from the bulb. At a distance of 3 meters from the bulb, I = 1.5 W/m^2. What is the approximate light intensity at a distance of 2 meters from the bulb?

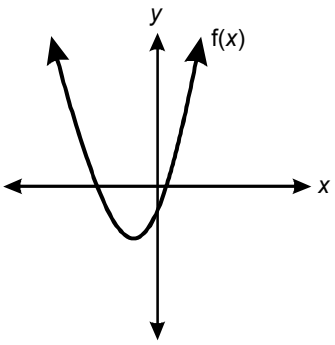
A. 2.3 W/m^2

B. 3.4 W/m^2

C. 6.8 W/m^2

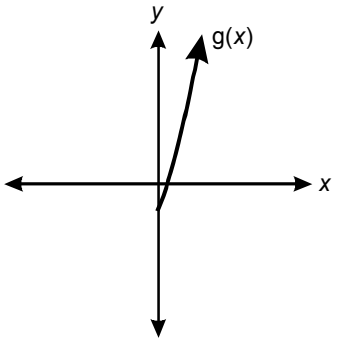
D. 13.5 W/m^2

39. **Use the graph below to answer the question that follows.**

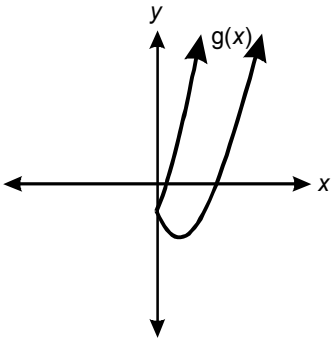


Based on the graph of f(x) above, which of the following is the graph of g(x) = $|f(x)|$?

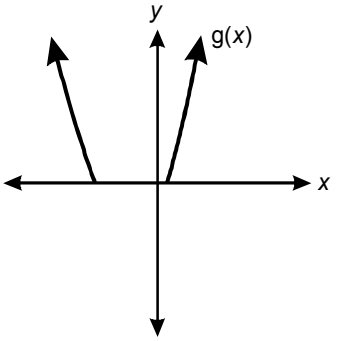
A.



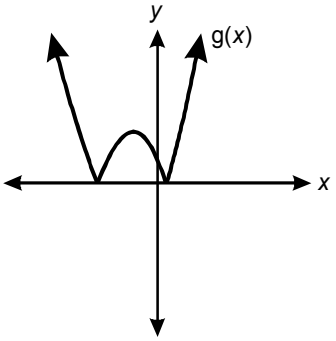
B.



C.



D.



40. **Use the equation below to answer the question that follows.**

$$\frac{x^2 - 2x - 15}{x^2 - 9} = 0$$

Which of the following is the complete solution to the equation above?

A. $x = \pm 3$

B. $x = 5$

C. $x = 5$, or $x = -3$

D. $x = 5$, or $x = \pm 3$

41. **Use the formula below to answer the question that follows.**

$$L = 10 \log \frac{I}{I_0}$$

Loudness of sound is measured in decibels according to the formula above, where I represents the intensity of the sound and I_0 is the intensity of the quietest sound detectable by the human ear. If sound intensity is proportional to wattage and a 30-watt stereo produces 100 decibels of sound, how many decibels are produced by a 3,000-watt professional sound system?

A. 102 dB

B. 120 dB

C. 1,000 dB

D. 10,000 dB

42. Carbon 14 decays exponentially with a half-life of 5750 years, the time it takes for one half of the carbon 14 to decay. If C_0 is the original concentration of carbon 14, which of the following functions correctly approximates $C(t)$, the quantity remaining as a function of time, t, in years?

A. $C(t) = C_0 e^{-0.000052t}$

B. $C(t) = C_0 e^{-0.00012t}$

C. $C(t) = C_0 e^{-2875t}$

D. $C(t) = C_0 e^{-5750t}$

43. **Use the expression below to answer the question that follows.**

$\ell n(x + 5)^4 - 2\ell n(x + 5)^3$

Which of the following is equivalent to the expression above?

A. $-2\ell n(x + 5)$

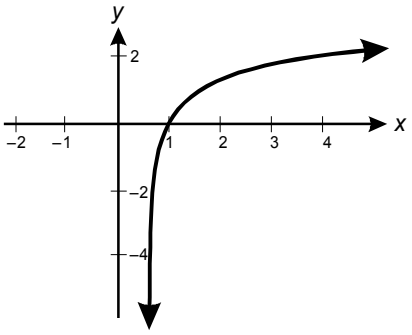
B. $\frac{2}{3}\ell n(x + 5)$

C. $[\ell n(x + 5)]^{-2}$

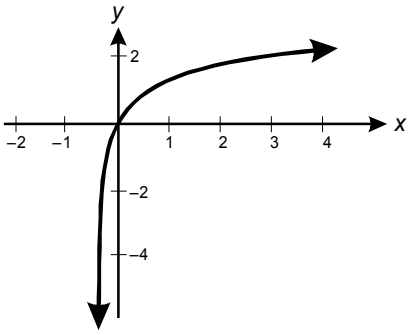
D. $[\ell n(x + 5)]^{\frac{2}{3}}$

44. Which of the following is the graph of the function $f(x) = \ell n(2x + 1)$?

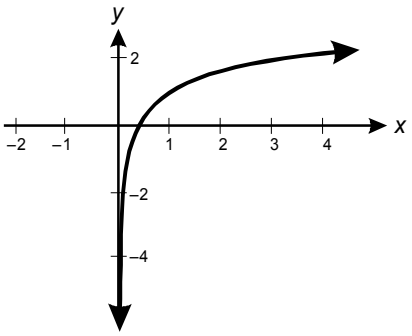
A.



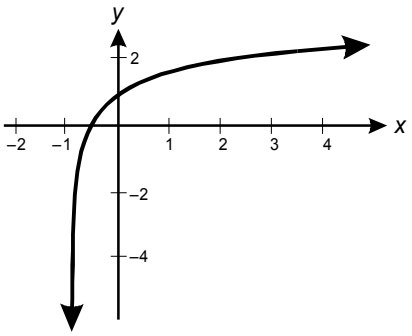
B.



C.



D.



45. Which of the following unit conversions is correct?

A. 10 cm = 0.01 m

B. $2\ mm^2 = 0.02\ cm^2$

C. $15\ m^2 = 150{,}000\ mm^2$

D. $8\ cm^3 = 80{,}000\ dm^3$

46. **Use the table below to answer the question that follows.**

Abbreviation	Meaning
$\frac{m}{t}$	miles per trip
$\frac{m}{g}$	miles per gallon of gasoline
$\frac{d}{g}$	dollars per gallon of gasoline
$\frac{d}{h}$	dollars per hour earned working

The table above defines a set of quantities related to a person working to pay for a trip. Which of the following expressions represents the number of hours that would have to be worked to earn enough money to pay for the gasoline for the trip?

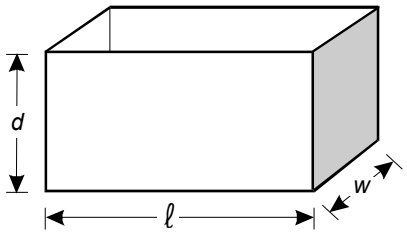
A. $\frac{\left(\frac{m}{t}\right)\left(\frac{m}{g}\right)}{\left(\frac{d}{g}\right)\left(\frac{d}{h}\right)}$

B. $\frac{\left(\frac{m}{t}\right)\left(\frac{d}{g}\right)}{\left(\frac{m}{g}\right)\left(\frac{d}{h}\right)}$

C. $\frac{\left(\frac{m}{g}\right)\left(\frac{d}{h}\right)}{\left(\frac{m}{t}\right)\left(\frac{d}{g}\right)}$

D. $\frac{\left(\frac{d}{g}\right)\left(\frac{d}{h}\right)}{\left(\frac{m}{t}\right)\left(\frac{m}{g}\right)}$

47. **Use the diagram below to answer the question that follows.**



The container above has length ℓ, width w, depth d, and volume V. A new container has a similar design except that the length is quadrupled, the width is doubled, and the depth is cut in half. Which of the following is equivalent to the volume of the new container?

A. $\frac{V}{2}$

B. V

C. $4V$

D. $8V$

48. A cylindrical can is measured to have a base radius of 5 cm and a height of 15 cm, both rounded to the nearest whole centimeter. The volume of the can is calculated based on these measurements. What is the maximum possible error in the calculated volume of the can?

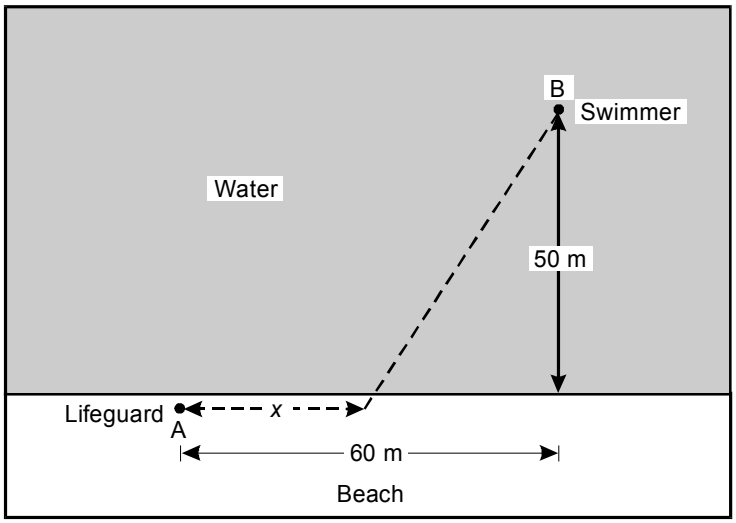
A. 232.7 cm^3

B. 255.6 cm^3

C. 275.3 cm^3

D. 294.9 cm^3

49. **Use the diagram below to answer the question that follows.**



A lifeguard at point *A* needs to reach a swimmer at point *B* as quickly as possible. The lifeguard runs along the beach a distance, *x*, at 3.5 meters per second. The lifeguard then swims along the diagonal dotted line at 1.5 meters per second. Which of the following expressions should be minimized to find the distance, *x*, that the lifeguard should run to reach the swimmer in the least amount of time?

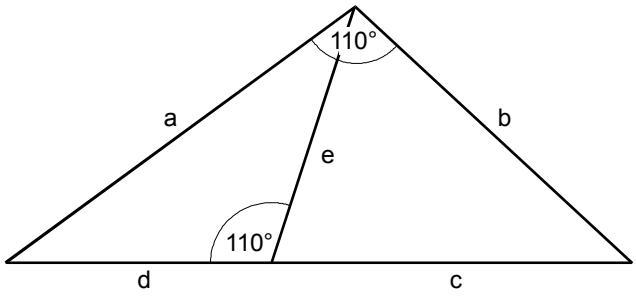
A. $\frac{60 - x}{3.5} + \frac{50}{1.5}$

B. $\frac{60 - x}{3.5} + \frac{(x - 60)^2 + 50^2}{1.5}$

C. $\frac{x}{3.5} + \frac{\sqrt{60^2 + 50^2}}{1.5}$

D. $\frac{x}{3.5} + \frac{\sqrt{(60 - x)^2 + 50^2}}{1.5}$

50. **Use the diagram below to answer the question that follows.**



Which of the following proportions can be deduced from the diagram above?

A. $\frac{d}{c} = \frac{a}{b}$

B. $\frac{d}{e} = \frac{b}{a}$

C. $\frac{e}{d} = \frac{c + d}{a}$

D. $\frac{a}{e} = \frac{c + d}{b}$

51. Given a line *m* and two points *A* and *B* not on *m*, there are infinitely many lines that pass through:

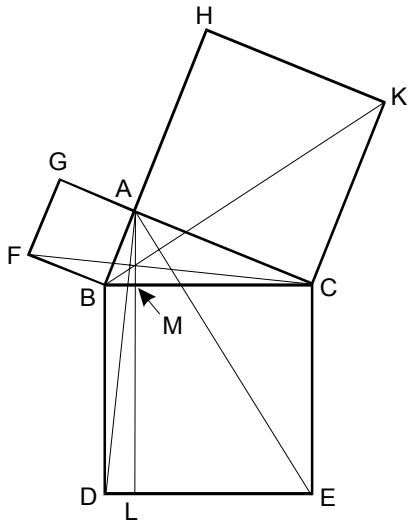
A. point *A* and are not parallel to *m*.

B. point *A* and are parallel to *m*.

C. points *A* and *B* and are not parallel to *m*.

D. points *A* and *B* and are parallel to *m*.

52. **Use the diagram below to answer the question that follows.**



The diagram above appears as part of a proof of the Pythagorean theorem as presented in Euclid's *Elements*. Which of the following best describes the conclusion of the proof?

A. the measure of $\angle BAC = 90°$

B. $BA^2 + BC^2 = AC^2$

C. Area of $\triangle BAC = \frac{1}{2}(BC)(MA)$

D. Area of $\square FGAB$ + Area of $\square HKCA$ = Area of $\square BCED$

53. Two sides of a right triangle have lengths of 6 and 8 units respectively. Which of the following could be the length of the third side?

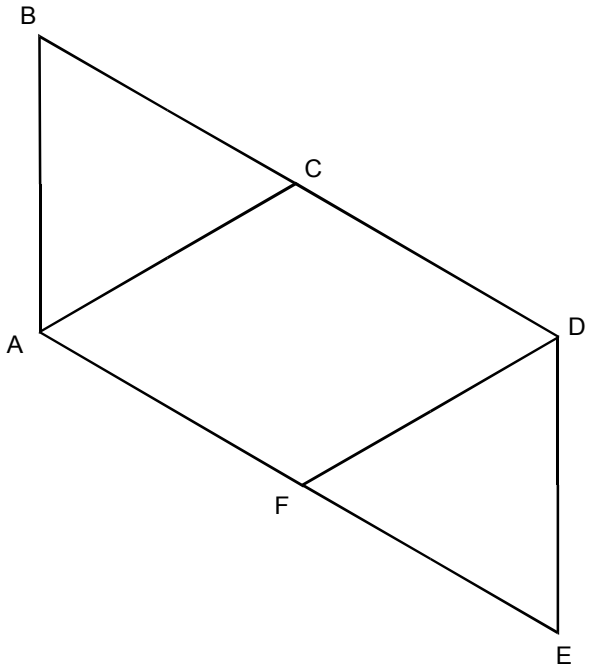
A. 7 units

B. 12 units

C. $2\sqrt{7}$ units

D. $4\sqrt{7}$ units

Use the diagram below to answer the two questions that follow.



54. If $\overline{AC}$ is parallel to $\overline{FD}$, which of the following triangle similarity relationships is sufficient to prove that $\overline{BD}$ is parallel to $\overline{AE}$?

A. $\triangle ABC \sim \triangle FDE$

B. $\triangle ABC \sim \triangle EFD$

C. $\triangle ABC \sim \triangle DFE$

D. $\triangle ABC \sim \triangle EDF$

55. If $\angle ABC \cong \angle FED$, which of the following additional conditions are sufficient to prove that $\triangle ABC \cong \triangle FED$?

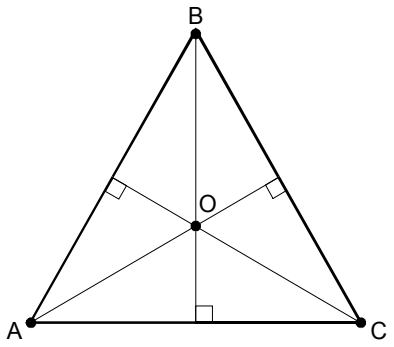
A. $\overline{AB} \cong \overline{FE}$ and $\overline{BC} \cong \overline{ED}$

B. $\overline{AB} \cong \overline{ED}$ and $\overline{AC} \cong \overline{FD}$

C. $\overline{AC} \cong \overline{FD}$ and $\overline{BC} \cong \overline{FE}$

D. $\overline{AB} \cong \overline{BC}$ and $\overline{FD} \cong \overline{DE}$

56. **Use the diagram below to answer the question that follows.**



$\triangle ABC$ above is an equilateral triangle. The three altitudes of $\triangle ABC$ meet at point O. If the length of $\overline{AB}$ is 2, which of the following is the length of $\overline{AO}$?

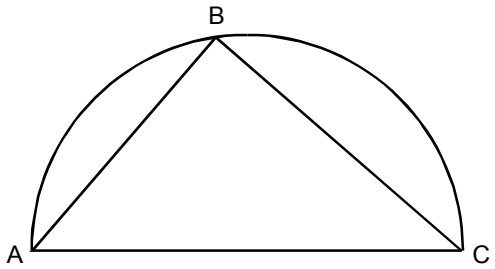
A. $\frac{\sqrt{3}}{3}$

B. $\frac{2\sqrt{3}}{3}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{3\sqrt{3}}{4}$

57. **Use the diagram below to answer the question that follows.**



Triangle *ABC* is inscribed in a semicircle with a diameter of 4 units. Line segment *BC* has a length of 3 units. What is the length of line segment *AB*?

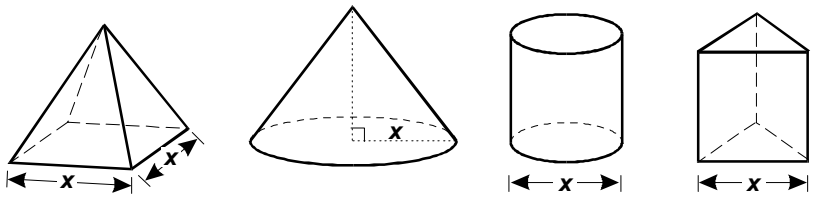
A. 2 units

B. 2.5 units

C. $\sqrt{5}$ units

D. $\sqrt{7}$ units

58. **Use the diagram below to answer the question that follows.**



The four solids shown above all have the same height. The square-based pyramid has base side length equal to *x*. The cone has base radius equal to *x*. The cylinder has base diameter equal to *x*. The equilateral triangle-based prism has base side length equal to *x*. Which solid has the greatest volume?

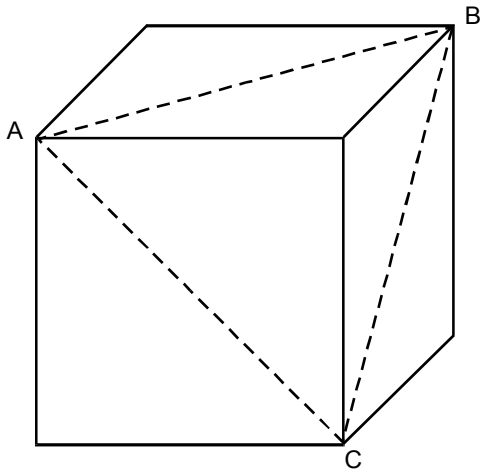
A. pyramid

B. cone

C. cylinder

D. prism

59. **Use the diagram below to answer the question that follows.**



The cube above has a volume equal to *V*. What is the perimeter of triangle *ABC*?

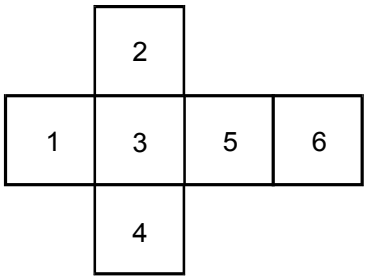
A. $3\sqrt[3]{V}$

B. $3\sqrt{2V}$

C. $3V\sqrt{2}$

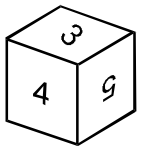
D. $3\sqrt{2}\sqrt[3]{V}$

60. **Use the diagram below to answer the question that follows.**

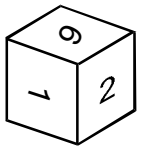


Which of the following cubes results from folding the pattern above?

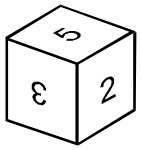
A.



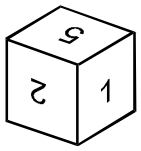
B.



C.



D.



61. **Use the diagram below to answer the question that follows.**

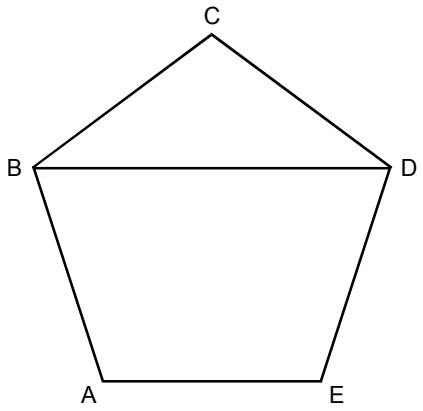


Figure *ABCDE* above is a regular pentagon. What is the measure of angle *ABD*?

A. 54°

B. 60°

C. 63°

D. 72°

62. The equation $x^2 + 2x + y^2 - 6y + 10 = 0$ represents which of the following?

A. a point

B. a circle

C. an ellipse

D. a hyperbola

63. Which of the following points is on the intersection of the *x*-*y* plane and the *y*-*z* plane?

    A. (0, 5, 0)

    B. (1, 1, 1)

    C. (2, 4, 0)

    D. (3, 0, 6)

64. Every point (*a*, *b*) on a graph is replaced by the point (*b*, *a*). The resulting graph is a reflection of the original graph across the:

    A. *x*-axis.

    B. *y*-axis.

    C. line $y = x$.

    D. origin.

65. Which of the following statements is valid in spherical geometry?

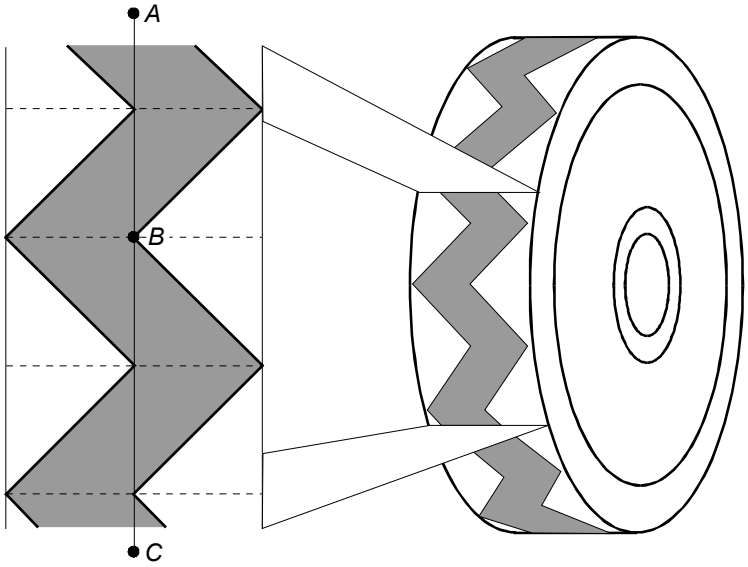
    A. Through any two points there is exactly one line.

    B. When two lines intersect, vertical angles are equal.

    C. The sum of the angles of any triangle is equal to 180 degrees.

    D. Through a point outside a given line, exactly one line can be drawn that does not intersect the given line.

Use the tire tread design below to answer the two questions that follow.



66. Under which of the following transformations will the design remain unchanged?

A. dilation with respect to point *B*

B. 180° rotation around point *B*

C. reflection across $\overline{AC}$

D. glide-reflection along $\overline{AC}$

67. The tire runs over a nail. If the nail is equally likely to hit all parts of the tread design, which of the following best approximates the probability that the nail will hit the raised (shaded) part of the tread?

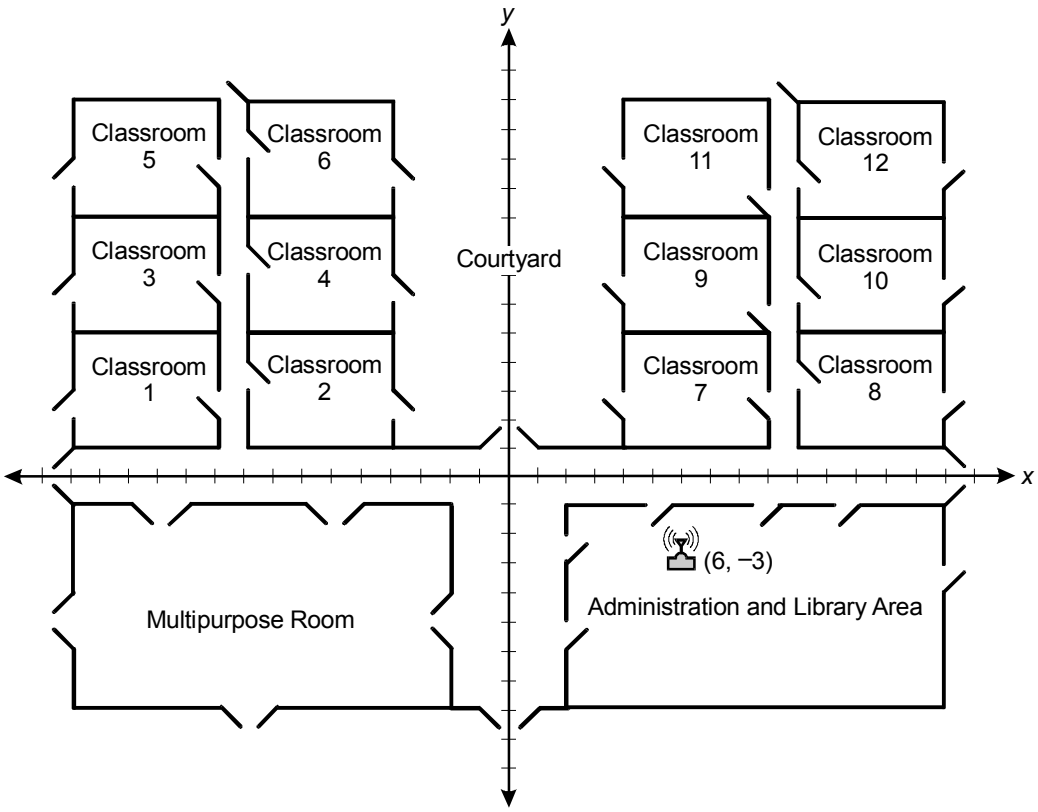
A. 0.3

B. 0.4

C. 0.5

D. 0.6

Use the diagram below to answer the two questions that follow.



68. A school building is placed on a set of coordinate axes, as shown in the diagram above, with one unit equal to five feet. A wireless computer network transmitter is located at the point (6, –3). The transmission signal has a reliable range of 50 feet in all directions. Which of the following inequalities represents the region of reliable coverage?

A. $(x + 6)^2 + (y – 3)^2 \leq 10$

B. $(x – 6)^2 + (y + 3)^2 \leq 10$

C. $(x + 6)^2 + (y – 3)^2 \leq 100$

D. $(x – 6)^2 + (y + 3)^2 \leq 100$

69. A student sits in five distinct randomly chosen classrooms over the course of a day. What is the probability that all five classrooms are located in the same quadrant?

A. $10 \div \frac{12!}{5!}$

B. $10 \div \frac{12!}{7! \bullet 5!}$

C. $12 \div \frac{12!}{5!}$

D. $12 \div \frac{12!}{7! \bullet 5!}$

70. Each of the numbers in a normally distributed data set is multiplied by two. How does this transformation affect the mean, the median, and the shape of the frequency distribution curve?

A. The mean doubles, but the median and the shape stay the same.

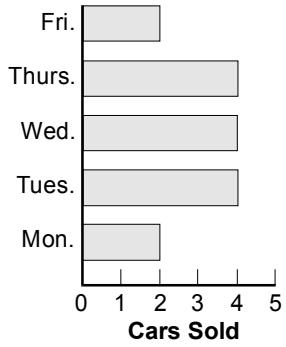
B. The mean and the median double, and the shape becomes wider.

C. The median doubles, but the mean and the shape stay the same.

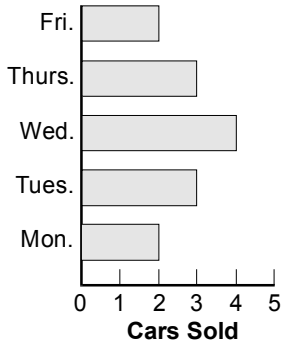
D. The median and the mean stay the same, but the shape becomes wider.

71. According to which of the following horizontal bar charts was the median number of cars sold per day equal to the mean number of cars sold per day?

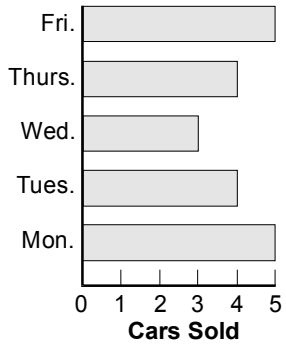
A.



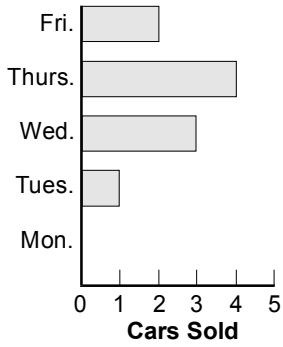
B.



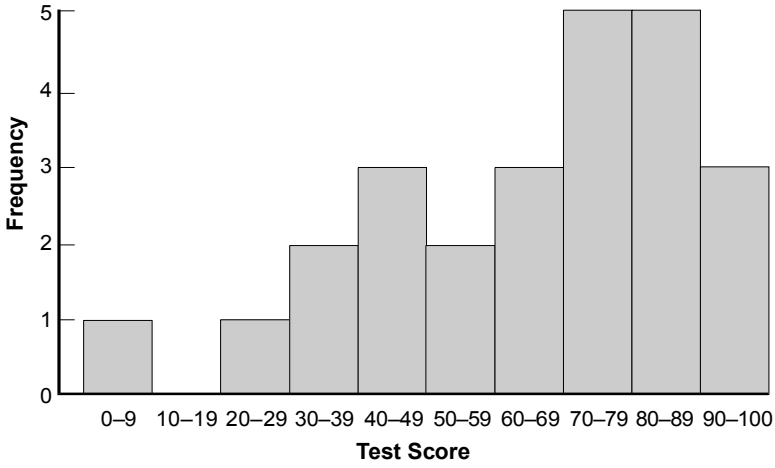
C.



D.



72. **Use the histogram below to answer the question that follows.**



Twenty-five students took a test. Their scores are represented by the histogram above. Which of the following scores could be in the 60th percentile?

A. 16

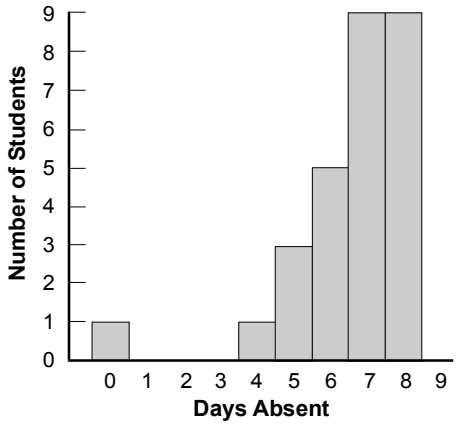
B. 41

C. 61

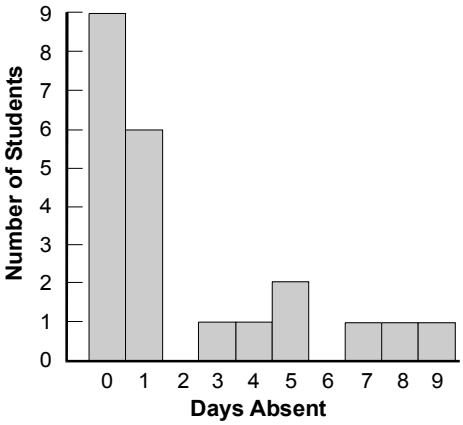
D. 76

73. A school administrator graphs the distribution of the number of days absent for students in four homerooms. Which of the following distributions has the smallest standard deviation of days absent?

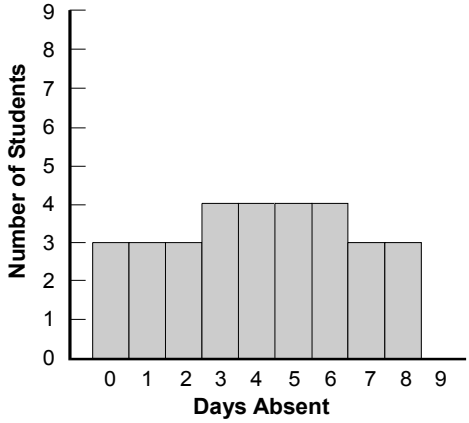
A.



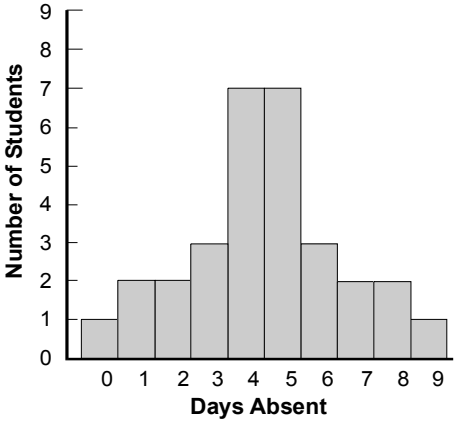
B.



C.

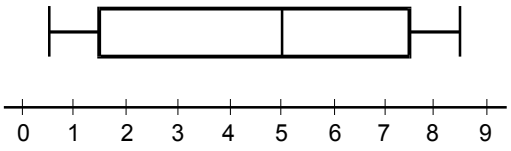


D.

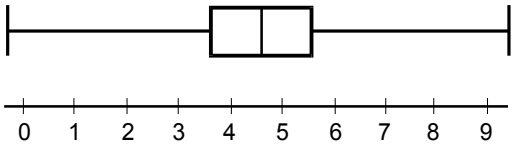


74. Which of the following box plots represents the largest range to interquartile range ratio?

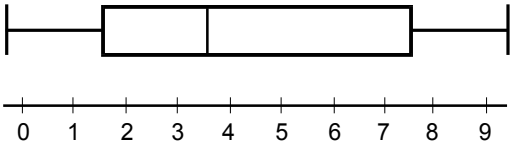
A.



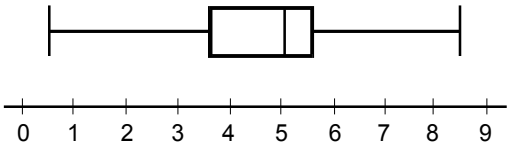
B.



C.



D.



75. **Use the information below to answer the question that follows.**

Linear Regression for Town A	**Linear Regression for Town B**
$y = ax + b$	$y = ax + b$
a = 0.4694308286	a = 3.8574694345
b = 7.738425386	b = 2.396738264
r = 0.9839264918	r = 0.0838567426

Students use a statistics program to study the populations of two towns. They enter twenty years of population data. The population in thousands is represented by *y*. The year is represented by *x*. The program generates the above linear regression results. Which of the following conclusions can be reached from these results?

A. The model for Town A is a better predictor of population than the model for Town B.

B. The population was initially smaller in Town B than in Town A.

C. The model for Town A predicted a faster growth rate than the model for Town B.

D. The population growth rate was accelerating in Town B and decelerating in Town A.

76. A government wants to make a rough calculation of the number of coins that are in circulation. In January they put an invisible marker on 10,000 of the coins and released them into circulation. In June they randomly collected 5,000 coins. Of the 5,000 coins collected, 37 were marked. Which of the following proportions should they solve for *T*, the total number of coins in circulation?

A. $\frac{37}{5{,}000} = \frac{T}{10{,}000}$

B. $\frac{37}{5{,}000} = \frac{T}{15{,}000}$

C. $\frac{37}{5{,}000} = \frac{10{,}000}{T}$

D. $\frac{37}{5{,}000} = \frac{15{,}000}{T}$

77. A car manufacturer claims that its new model gets 55 miles per gallon of gasoline on the highway. A consumer organization uses hypothesis testing to evaluate this claim. Let μ be the mean highway fuel efficiency of the new model in miles per gallon. What should the consumer organization use as H_0, the null hypothesis, and H_1, the alternate hypothesis?

A. $H_0: \mu < 55$
$H_1: \mu = 55$

B. $H_0: \mu > 55$
$H_1: \mu = 55$

C. $H_0: \mu = 55$
$H_1: \mu < 55$

D. $H_0: \mu = 55$
$H_1: \mu > 55$

78. **Use the tables below to answer the question that follows.**

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
City 1	2	4	5	5	4	6	7
City 2	8	6	5	5	3	2	1

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
City 3	−8	−6	−6	−4	−3	−2	−1
City 4	1	2	3	4	6	6	8

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
City 5	5	5	7	3	3	8	4
City 6	7	4	3	5	4	5	2

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
City 7	8	6	6	4	3	2	1
City 8	−1	−2	−3	−4	−6	−6	−8

The tables above show the daily high temperatures in Celsius for four pairs of cities during a particular week. For which pair of cities are the temperatures negatively correlated?

A. City 1 and City 2

B. City 3 and City 4

C. City 5 and City 6

D. City 7 and City 8

79. A car rental company has compact disc (CD) players in 85% of its cars. The CD players are randomly distributed throughout the fleet of cars. If a person rents four cars, what is the probability that at least three of them will have CD players?

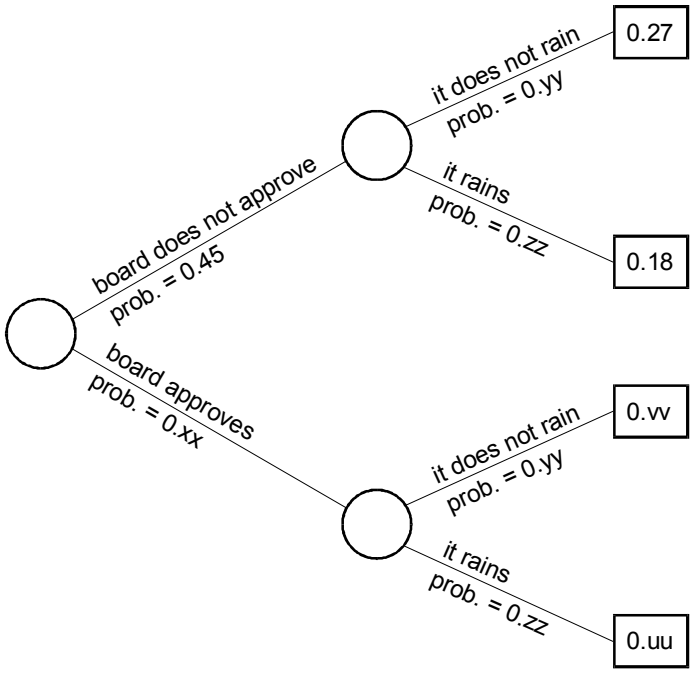
A. 15%

B. 37%

C. 61%

D. 89%

80. **Use the diagram below to answer the question that follows.**



A teacher calculates the probability of success for an outdoor field trip. For the trip to be a success, first, the school board must approve the plan, and second, it must not rain. According to the tree diagram above, what is the probability of a successful field trip?

A. 0.18

B. 0.22

C. 0.27

D. 0.33

81. A bag contains 80 marbles: 20 red, 20 blue, 20 yellow, and 20 green. What is the probability that the first 4 marbles randomly selected, without replacement, are all different colors?

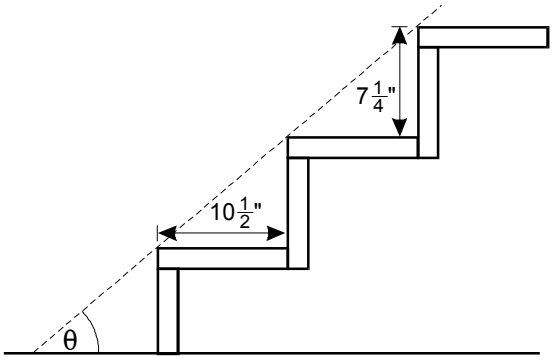
A. 0.42%

B. 1.56%

C. 9.38%

D. 10.12%

82. **Use the diagram below to answer the question that follows.**



Each step of the staircase above has a rise of $7\frac{1}{4}$ inches and a run of $10\frac{1}{2}$ inches. What is θ, the angle of the staircase to the floor, rounded to the nearest whole degree?

A. 35°

B. 44°

C. 46°

D. 55°

83. Which of the following angles lies in the second quadrant of the polar coordinate system?

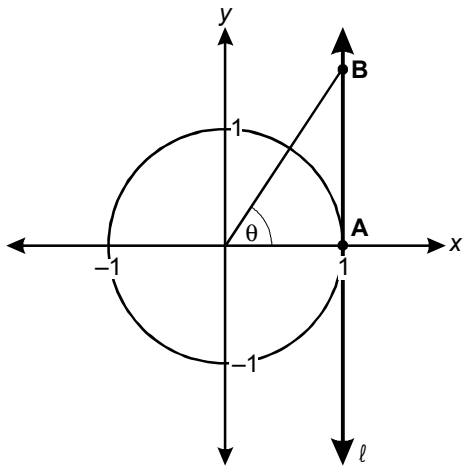
A. $-145°$

B. $541°$

C. 101π radians

D. $\frac{602\pi}{3}$ radians

84. **Use the diagram below to answer the question that follows.**



If line ℓ is vertical, which of the following expressions represents the length of $\overline{AB}$?

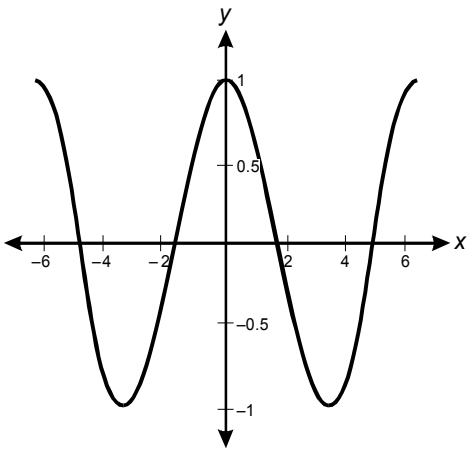
A. $\cos\theta$

B. $\sec\theta$

C. $\sin\theta$

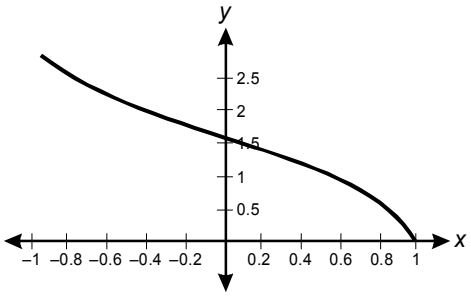
D. $\tan\theta$

85. **Use the graph below to answer the question that follows.**

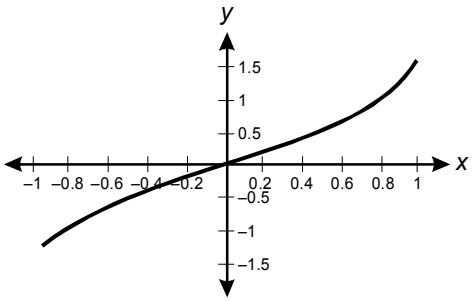


Given the above graph of a trigonometric relation, which of the following graphs represents its inverse?

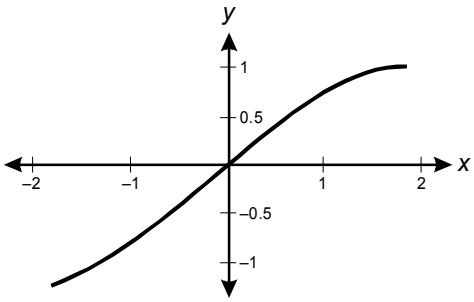
A.



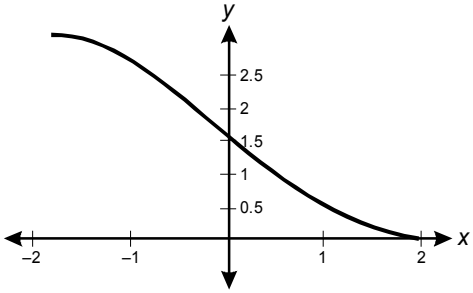
B.



C.



D.



86. **Use the function below to answer the question that follows.**

$$f(x) = \frac{x^3 - 2x^2 + 3}{4x^3 + 5x - 1}$$

Which of the following is the limit of $f(x)$ as x approaches infinity?

A. -3

B. 0

C. $\frac{1}{4}$

D. $\frac{1}{2}$

87. Which of the following functions is continuous for all real values of x?

A. $f(x) = \frac{x^2 - 9}{x - 3}$

B. $f(x) = \frac{x^2 - 9}{\sqrt{x - 3}}$

C. $f(x) = \begin{cases} x^2 + 2 \text{ for } x \leq 1 \\ x + 2 \text{ for } x > 1 \end{cases}$

D. $f(x) = \begin{cases} x + 2 \text{ for } x \leq 1 \\ x - 2 \text{ for } x > 1 \end{cases}$

88. If functions f(x) and g(x) are continuous at $x = a$, then which of the following functions could be discontinuous at $x = a$?

A. $h(x) = f(x)g(x)$

B. $h(x) = f(x) - g(x)$

C. $h(x) = f(x) + g(x)$

D. $h(x) = \frac{f(x)}{g(x)}$

89. A bicyclist pedaled up a 2-mile-long hill averaging 4 mph. Upon reaching the top, the bicyclist headed down a 6-mile descent averaging 26 mph. What was the bicyclist's approximate average speed?

A. 7.3 mph

B. 10.9 mph

C. 15 mph

D. 20.5 mph

90. **Use the equation below to answer the question that follows.**

$y = 5 + x \sin x + \cos x$

The equation above is a solution to which of the following differential equations?

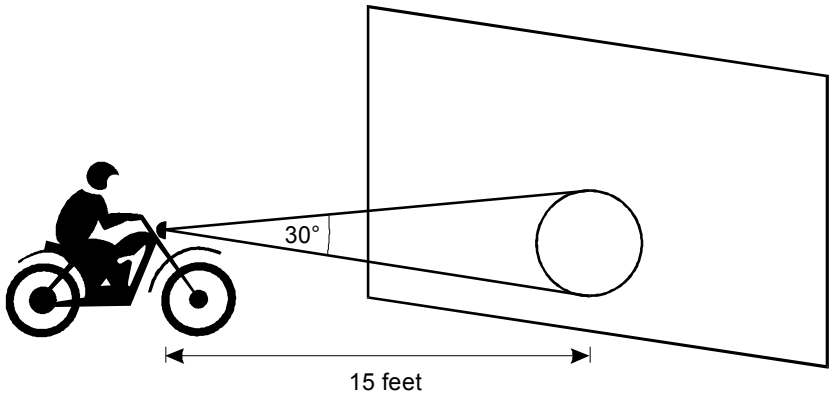
A. $\frac{dy}{dx} = \cos x$

B. $\frac{dy}{dx} = x \cos x$

C. $\frac{dy}{dx} = \cos x - \sin x$

D. $\frac{dy}{dx} = 2 \sin x - x \cos x$

91. **Use the diagram below to answer the question that follows.**



A motorcycle headlight shines a cone of light with a 30° angle of dispersion as shown above. The light creates a bright circle on a wall in front of the motorcycle. The motorcycle is moving toward the wall at 10 feet per second. What is the approximate instantaneous rate of change of the area of the bright circle when the motorcycle is 15 feet from the wall?

A. –7 square feet per second

B. –32 square feet per second

C. –68 square feet per second

D. –314 square feet per second

92. What is the slope of the line tangent to the graph of $y = (6 - x^2)\left(\frac{1}{2}x^2 + \frac{1}{x}\right)$ at $x = 1$?

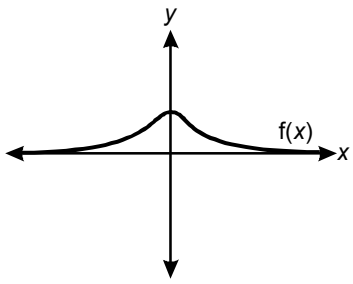
A. –3

B. –2

C. 0

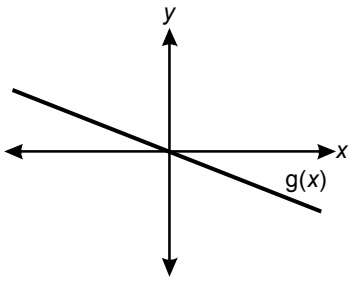
D. 2

93. **Use the graph below to answer the question that follows.**

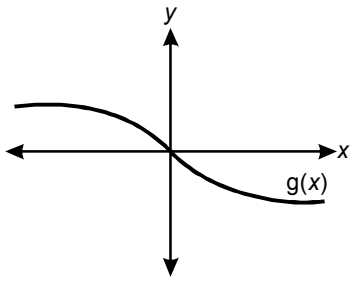


Based on the graph of f(x) above, which of the following graphs best approximates g(x) = f′(x)?

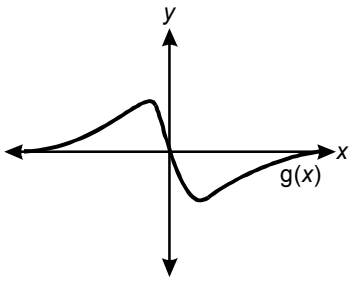
A.



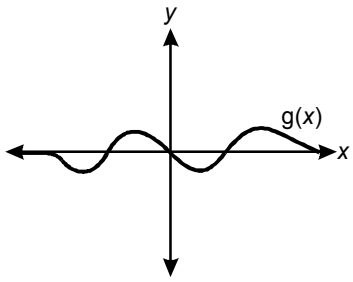
B.



C.



D.



94. Which of the following definite integrals represents the length of the curve defined by f(x) = x^2 on the x-interval (0, 3)?

A. $\int_0^3 \sqrt{1 + 2x}\, dx$

B. $\int_0^3 \sqrt{1 + x^2}\, dx$

C. $\int_0^3 \sqrt{1 + 4x^2}\, dx$

D. $\int_0^3 \sqrt{1 + x^4}\, dx$

95. **Use the function below to answer the question that follows.**

$c(t) = 1.2e^{0.14t}$

The manager of a new water treatment plant estimates that the plant's capacity will grow according to the function above, where c represents capacity in millions of gallons per year and t represents the number of years since the plant opened. Based on this estimate, which of the following approximates the total number of gallons of water that will be treated in the first five years?

A. 2.4 million gallons

B. 8.7 million gallons

C. 9.3 million gallons

D. 17.3 million gallons

96. **Use the table below to answer the question that follows.**

Time	Speed (mph)
12:00	0
12:02	20
12:04	45
12:06	40
12:08	15
12:10	0

The table above shows the speed of a car recorded at two-minute intervals during a ten-minute trip. A mathematician employs the left-endpoint rectangle method to estimate the total distance traveled. Which of the following statements correctly predicts the accuracy of this technique?

A. The technique will consistently underestimate the distance over the entire course of the trip.

B. The technique will underestimate the distance for the first half of the trip and overestimate the distance for the second half of the trip.

C. The technique will overestimate the distance for the first half of the trip and underestimate the distance for the second half of the trip.

D. The technique will consistently overestimate the distance over the entire course of the trip.

97. Which of the following equations results from solving the differential equation $\frac{dy}{dx} + y^3 \cos x = 0$?

A. $\frac{y^{-2}}{2} = \cos(x) + c$

B. $\frac{y^2}{2} = -\sin(x) + c$

C. $-\frac{y^{-2}}{2} = -\sin(x) + c$

D. $-\frac{y^2}{2} = \cos(x) + c$

98. Which of the following statements holds for all 2 by 2 matrices A, B, and C?

A. $AB = BA$

B. $A(B + C) = AB + AC$

C. If $AB = 0$, then $A = 0$ or $B = 0$.

D. If $AB = AC$, then $B = C$.

99. A person invests $10,000 at a 5% annual interest rate compounded quarterly. Solving which of the following equations yields t, the doubling time in years?

A. $e^{0.0125t} = 2$

B. $e^{0.05t} = 2$

C. $(1.0125)^{4t} = 2$

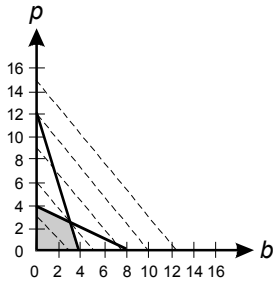
D. $(1.05)^{4t} = 2$

100. **Use the table below to answer the question that follows.**

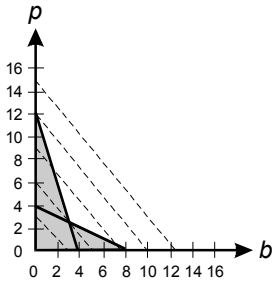
Production Parameters per 1,000			
product type	**profit (dollars)**	**prep time (hours)**	**bake time (hours)**
bagels (*b*)	60	2	3
pretzels (*p*)	50	4	1

The manager of a commercial bakery uses the table above to optimize the profitability of the bakery. The bakery's preparation room can operate for up to 16 hours per day and its baking facility can run for up to 12 hours per day. If *b* represents thousands of bagels and *p* represents thousands of pretzels, which of the following graphs represents the linear programming problem?

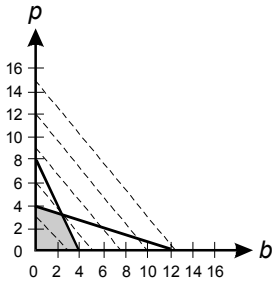
A.



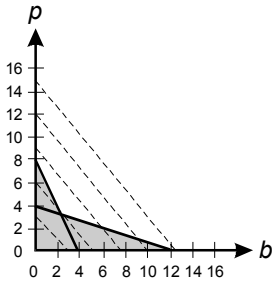
B.



C.



D.



DIRECTIONS FOR THE OPEN-RESPONSE ITEM ASSIGNMENTS

This section of the test consists of two open-response item assignments that appear on the following pages. You will be asked to prepare a written response of approximately 1–2 pages for each assignment. You should use your time to plan, write, review, and edit your response for each assignment.

For each assignment, read the topic and directions carefully before you begin to work. Think about how you will organize your response. You may use any blank space in this test booklet to make notes, write an outline, or otherwise prepare your response.

As a whole, your response to each assignment must demonstrate an understanding of the knowledge of the field. In your response to each assignment, you are expected to demonstrate the depth of your understanding of the subject area by applying your knowledge rather than by merely reciting factual information.

Your response to each assignment will be evaluated based on the following criteria.

- **PURPOSE:** the extent to which the response achieves the purpose of the assignment
- **SUBJECT KNOWLEDGE:** appropriateness and accuracy in the application of subject knowledge
- **SUPPORT:** quality and relevance of supporting evidence
- **RATIONALE:** soundness of argument and degree of understanding of the subject area

The open-response item assignments are intended to assess subject knowledge. Your responses must be communicated clearly enough to permit valid judgment of the evaluation criteria by scorers. Your responses should be written for an audience of educators in this field. The final version of each response should conform to the conventions of edited American English. Your responses should be your original work, written in your own words, and not copied or paraphrased from some other work.

Be sure to write about the assigned topics. Please write legibly. You may not use any reference materials during the test. Remember to review your work and make any changes you think will improve your responses.

OPEN-RESPONSE ITEM ASSIGNMENT #1

Use the information below to complete the exercise that follows.

The table below shows the annual profit in dollars of a start-up company during its first three years of operation.

Year Number	Annual Profit in Dollars
1	500
2	5,000
3	50,000

Using your knowledge of functions and data analysis, prepare a response in which you analyze the situation. In your response:

- model annual profit as a function of year number using an exponential function of the form $E(n) = ab^n$;
- model annual profit as a function of year number using a quadratic function of the form $Q(n) = an^2 + bn + c$ by deriving and solving a system of linear equations in a, b, and c;
- use each model to predict the annual profit of the company's fourth year; and
- if $Q(n)$ is the correct model, calculate the percent error of the annual profit predicted by $E(n)$ for the fourth year.

Be sure to show your work and explain the reasoning you use in analyzing and solving this problem.

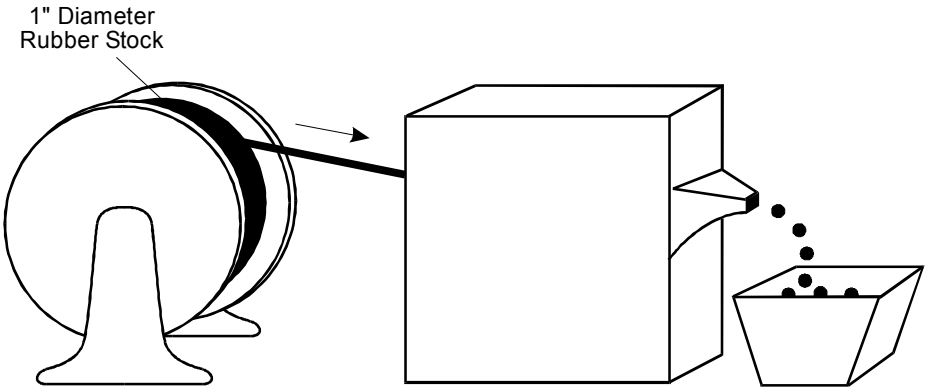
OPEN-RESPONSE SHEET—ASSIGNMENT #1

OPEN-RESPONSE SHEET—ASSIGNMENT #1

OPEN-RESPONSE ITEM ASSIGNMENT #2

Use the information below to complete the exercise that follows.

The machine shown below manufactures various-sized solid spherical rubber balls from a continuously fed 1" diameter cylindrical rubber stock. For each run, the balls produced are uniform in size. The machine manufactures three balls per second. The total volume of rubber stock fed into the machine is exactly equal to the combined volume of rubber balls manufactured.



Using your knowledge of three-dimensional geometry and functions, prepare a response in which you analyze this situation. In your response:

- calculate the volume $V_b(d)$, in cubic inches, of one rubber ball as a function of d, the diameter of the ball;
- calculate the volume V_s, in cubic inches, of one linear foot of the rubber stock;
- derive the function $s(d)$, where s is the speed in feet per second of the rubber feed and d is the diameter in inches of each rubber ball; and
- plot a graph of the function $s(d)$ for values of d from 0.5" to 2.5".

Be sure to show your work and explain the reasoning you use in analyzing and solving this problem.

OPEN-RESPONSE SHEET—ASSIGNMENT #2

OPEN-RESPONSE SHEET—ASSIGNMENT #2

PRACTICE TEST RESULTS

PRACTICE TEST RESULTS OVERVIEW

The practice test provides valuable information regarding your preparedness for the MTEL Mathematics (09) test. In this section, you will find information and tools to help you determine your preparedness on the various sections of the test.

Multiple-Choice Questions

An Answer Key Worksheet is provided to assist you in evaluating your multiple-choice responses. The worksheet contains five columns. The first column indicates the multiple-choice question number, the second column indicates the objective to which the item was written, and the third column indicates the correct response. The fourth and fifth columns are for your use in calculating the number of multiple-choice questions you answered correctly or incorrectly.

An Evaluation Chart for the multiple-choice questions is also provided to help you assess which content covered by the test objectives may require additional study.

Open-Response Items

Evaluation Information, Sample Responses and Analyses, as well as a Scoring Rubric are provided for these items. You may wish to refer to this information when evaluating your practice test responses.

Total Test

Practice Test Score Calculation information is provided to help you estimate your score on the practice test. Although you cannot use this practice test to precisely predict how you might score on an official MTEL Mathematics (09) test, you may be able to determine your degree of readiness to take an MTEL test at an operational administration. No passing score has been determined for the practice test.

MULTIPLE-CHOICE QUESTION ANSWER KEY WORKSHEET

Question Number	Objective Number	Correct Response	Your Response	
			Correct?	Incorrect?
1	0001	B		
2	0001	D		
3	0001	D		
4	0001	A		
5	0001	C		
6	0003	A		
7	0002	B		
8	0013	A		
9	0002	D		
10	0002	C		
11	0002	B		
12	0002	C		
13	0003	A		
14	0003	B		
15	0003	A		
16	0003	D		
17	0004	C		
18	0004	B		
19	0004	D		
20	0004	C		
21	0005	C		
22	0005	C		
23	0005	D		
24	0005	C		
25	0006	C		
26	0006	B		
27	0006	B		
28	0006	C		
29	0007	B		
30	0007	A		
31	0007	C		
32	0007	A		
33	0008	A		
34	0008	D		

MULTIPLE-CHOICE QUESTION
ANSWER KEY WORKSHEET (continued)

Question Number	Objective Number	Correct Response	Your Response	
			Correct?	Incorrect?
35	0008	B		
36	0008	D		
37	0009	C		
38	0009	B		
39	0009	D		
40	0009	B		
41	0010	B		
42	0010	B		
43	0010	A		
44	0010	B		
45	0011	B		
46	0011	B		
47	0011	C		
48	0011	D		
49	0011	D		
50	0012	D		
51	0012	A		
52	0012	D		
53	0012	C		
54	0013	D		
55	0013	A		
56	0013	B		
57	0013	D		
58	0014	B		
59	0014	D		
60	0014	B		
61	0014	D		
62	0015	A		
63	0015	A		
64	0015	C		
65	0015	B		
66	0015	D		
67	0018	C		
68	0015	D		

MULTIPLE-CHOICE QUESTION
ANSWER KEY WORKSHEET (continued)

Question Number	Objective Number	Correct Response	Your Response	
			Correct?	Incorrect?
69	0023	D		
70	0016	B		
71	0016	D		
72	0016	D		
73	0016	A		
74	0016	B		
75	0017	A		
76	0017	C		
77	0017	C		
78	0017	A		
79	0018	D		
80	0018	D		
81	0018	D		
82	0019	A		
83	0019	D		
84	0019	D		
85	0019	A		
86	0020	C		
87	0020	C		
88	0020	D		
89	0020	B		
90	0021	B		
91	0021	C		
92	0021	A		
93	0021	C		
94	0022	C		
95	0022	B		
96	0022	B		
97	0022	C		
98	0023	B		
99	0023	C		
100	0023	A		

Count the number of multiple-choice questions you answered correctly:

_____ of 100 multiple-choice questions

MULTIPLE-CHOICE QUESTION PRACTICE TEST EVALUATION CHART

In the evaluation chart that follows, the multiple-choice questions are arranged in numerical order and by test objective. Check your responses against the correct responses provided to determine how many questions within each objective you answered correctly.

Subarea I: Number Sense and Operations

Objective 0001: Understand the structure of numeration systems and solve problems using integers, fractions, decimals, percents, ratios, and proportions.	
1B____ 2D_____ 3D_____ 4A_____ 5C_____	**_____/5**

Objective 0002: Understand the properties of real and complex numbers and the real and complex number systems.	
7B____ 9D_____ 10C_____ 11B_____ 12C_____	**_____/5**

Objective 0003: Understand the principles of number theory.	
6A____ 13A____ 14B_____ 15A_____ 16D_____	**_____/5**

Subarea I (Objectives 0001–0003) Total _____/15

MULTIPLE-CHOICE QUESTION
PRACTICE TEST EVALUATION CHART (continued)

Subarea II: Patterns, Relations, and Algebra

Objective 0004: Understand and use patterns to model and solve problems.	
17C____ 18B_____ 19D_____ 20C_____	_____/4

Objective 0005: Understand the properties of functions and relations.	
21C____ 22C_____ 23D_____ 24C_____	_____/4

Objective 0006: Understand the properties and applications of linear relations and functions.	
25C____ 26B_____ 27B_____ 28C_____	_____/4

Objective 0007: Understand the properties and applications of linear and abstract algebra.	
29B____ 30A_____ 31C_____ 32A_____	_____/4

Objective 0008: Understand the properties and applications of quadratic relations and functions.	
33A____ 34D_____ 35B_____ 36D_____	_____/4

Objective 0009: Understand the properties and applications of polynomial, radical, rational, and absolute value functions and relations.	
37C____ 38B_____ 39D_____ 40B_____	_____/4

Objective 10: Understand the properties and applications of exponential and logarithmic functions and relations.	
41B____ 42B_____ 43A_____ 44B_____	_____/4

Subarea II (Objectives 0004–0010) Total _____/28

MULTIPLE-CHOICE QUESTION
PRACTICE TEST EVALUATION CHART (continued)

Subarea III: Geometry and Measurement

Objective 0011: Understand the principles, concepts, and procedures related to measurement.	
45B____ 46B_____ 47C_____ 48D_____ 49D_____	_____/5

Objective 0012: Understand the axiomatic structure of Euclidean geometry.	
50D____ 51A_____ 52D_____ 53C_____	_____/4

Objective 0013: Prove theorems within the axiomatic structure of Euclidean geometry.	
8A____ 54D____ 55A_____ 56B_____ 57D_____	_____/5

Objective 0014: Apply Euclidean geometry to solve problems involving two- and three-dimensional objects.	
58B____ 59D_____ 60B_____ 61D_____	_____/4

Objective 0015: Understand the principles and properties of coordinate and transformational geometry and characteristics of non-Euclidean geometries.	
62A____ 63A_____ 64C_____ 65B_____ 66D_____ 68D_____	_____/6

Subarea III (Objectives 0011–0015) Total _____/24

MULTIPLE-CHOICE QUESTION
PRACTICE TEST EVALUATION CHART (continued)

Subarea IV: Data Analysis, Statistics, and Probability

Objective 0016: Understand the principles and concepts of descriptive statistics and their application to the problem-solving process.	
70B____ 71D_____ 72D_____ 73A_____ 74B_____	_____/5

Objective 0017: Understand the methods used in collecting and analyzing data.	
75A____ 76C____ 77C_____ 78A_____	_____/4

Objective 0018: Understand the fundamental principles of probability.	
67C____ 79D____ 80D_____ 81D_____	_____/4

Subarea IV (Objectives 0016–0018) Total _____/13

MULTIPLE-CHOICE QUESTION
PRACTICE TEST EVALUATION CHART (continued)

Subarea V: Trigonometry, Calculus, and Discrete Mathematics

Objective 0019: Understand the properties of trigonometric functions and identities.	
82A____ 83D_____ 84D_____ 85A_____	_____/4

Objective 0020: Understand the concepts of limit, continuity, and rate of change.	
86C____ 87C_____ 88D_____ 89B_____	_____/4

Objective 0021: Understand differential calculus.	
90B____ 91C_____ 92A_____ 93C_____	_____/4

Objective 0022: Understand integral calculus.	
94C____ 95B_____ 96B_____ 97C_____	_____/4

Objective 0023: Understand the principles of discrete/finite mathematics.	
69D____ 98B_____ 99C_____ 100A_____	_____/4

Subarea V (Objectives 0019–0023) Total _____/20

OPEN-RESPONSE ITEM EVALUATION INFORMATION

How Open-Response Items Are Scored

Open-response items are scored through a process called focused holistic scoring. Scorers judge the overall effectiveness of the response rather than individual aspects considered in isolation. Scorer judgments are based on the quality of the response, not on length or neatness. Responses must be long enough to cover the topic adequately and scorers must be able to read what is written.

How to Evaluate Your Practice Responses

On the following pages, you will find two "strong" and two "weak" sample responses. PLEASE DO NOT REVIEW THE SAMPLE RESPONSES UNTIL AFTER YOU HAVE WRITTEN YOUR OWN RESPONSE. When you do review the two "strong" and "weak" sample responses and analyses included here, please note the following points:

- ✓ For the purposes of the practice test, responses are identified as "strong" or "weak" rather than given a score point of 1–4.
- ✓ The responses identified as "strong" may contain flaws; however, these responses do demonstrate the performance characteristics of a "strong response."
- ✓ The two "strong" responses demonstrate the examinees' appropriate understanding and application of the subject matter knowledge. However, these responses do not necessarily reflect the full range of "correct answers" that would demonstrate an understanding of the subject matter.
- ✓ The "Analysis" accompanying each "strong" and "weak" response discusses the main attributes of the responses, but does not identify all flaws or strengths that may be present.

Compare your practice responses to the Sample Responses to determine whether your responses are more similar to the strong or weak responses. Also review the Analyses on those pages and the Scoring Rubric to help you better understand the characteristics of strong and weak responses. This evaluation will help you identify specific problems or weaknesses in your practice responses. Further information on scoring can be found in the Test Information Booklet and Faculty Guide at www.mtel.nesinc.com and at www.doe.mass.edu/mtel; select "FAQ," then "After the Test."

OPEN-RESPONSE ITEM
SCORING RUBRIC, SAMPLE RESPONSES, AND ANALYSES

Massachusetts Tests for Educator Licensure®
SCORING RUBRIC FOR SUBJECT TESTS

Performance Characteristics:

Purpose	The extent to which the response achieves the purpose of the assignment.
Subject Matter Knowledge	Accuracy and appropriateness in the application of subject matter knowledge.
Support	Quality and relevance of supporting details.
Rationale	Soundness of argument and degree of understanding of the subject matter.

Scoring Scale:

Score Point	Score Point Description
4	**The "4" response reflects a thorough knowledge and understanding of the subject matter.** • The purpose of the assignment is fully achieved. • There is a substantial, accurate, and appropriate application of subject matter knowledge. • The supporting evidence is sound; there are high-quality, relevant examples. • The response reflects an ably reasoned, comprehensive understanding of the topic.
3	**The "3" response reflects an adequate knowledge and understanding of the subject matter.** • The purpose of the assignment is largely achieved. • There is a generally accurate and appropriate application of subject matter knowledge. • The supporting evidence is adequate; there are some acceptable, relevant examples. • The response reflects an adequately reasoned understanding of the topic.
2	**The "2" response reflects a limited knowledge and understanding of the subject matter.** • The purpose of the assignment is partially achieved. • There is a limited, possibly inaccurate or inappropriate, application of subject matter knowledge. • The supporting evidence is limited; there are few relevant examples. • The response reflects a limited, poorly reasoned understanding of the topic.
1	**The "1" response reflects a weak knowledge and understanding of the subject matter.** • The purpose of the assignment is not achieved. • There is little or no appropriate or accurate application of subject matter knowledge. • The supporting evidence, if present, is weak; there are few or no relevant examples. • The response reflects little or no reasoning about or understanding of the topic.

U	**The response is unrelated to the assigned topic, illegible, primarily in a language other than English, not of sufficient length to score, or merely a repetition of the assignment.**
B	**There is no response to the assignment.**

FIRST SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #1

The formula for the annual profit is $E(n) = 50(10)^n$. This works for all years thus far. This model would predict that in the fourth year there would be an annual profit of $500,000. This can be shown by the equation:

$$E(n) = 50(10)^4$$
$$E(n) = \$500{,}000$$

There are no other formulas that work for this data. By using the method of differences, one can see that a quadratic formula would be impossible in these circumstances.

1	500		
		4500	
2	5000		40500
		45000	
3	50000		405000
		450000	
4	500000		4050000
		450000	
5	5000000		

This process would go on in this manner for thousands of differences. It is not practical or really possible to find a quadratic formula given the behavior of the data, therefore it can only be determined through the exponential function, $E_n = 50(10)^n$.

ANALYSIS FOR FIRST WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

This is an example of a weak response because it is characterized by the following:

Purpose: The purpose of the assignment is partially achieved. The candidate derives one of the functions and incorrectly assumes that the second model is impossible. The candidate then correctly uses the first model to predict the fourth year. Because the candidate does not use the second model, it is impossible to calculate the percent error as requested in the last part of the problem.

Subject Matter Knowledge: The statement, "There are no other formulas that work for this data," shows limited understanding of quadratics and mathematical modeling. The candidate is missing the fundamental concept that three distinct nonlinear points can always be uniquely fit with a quadratic function. The candidate's use of second differences is not relevant to solving this problem because the problem only specifies three data points. The candidate extrapolates the data using the first model and then attempts to fit the results to the second model. This shows a lack of understanding of the distinction between observed data and data predicted by mathematical modeling.

Support: Although the response lacks support for the derivation of the exponential function, it shows the calculation of the 4th year's profit using that function. The candidate fails to explain that the percent error could not be calculated without the quadratic model. This response is not made stronger by the verbal explanation because it is clear that the candidate has a limited understanding of the content.

Rationale: The candidate reaches the conclusion that a quadratic model is not possible by assuming an exponential pattern from the first three data points and inappropriately applying a difference method to determine that a quadratic pattern is impossible. In this way, the candidate shows a limited understanding of the mathematical modeling process in general, and of quadratic functions in particular. The candidate also fails to demonstrate the reasoning behind the solution to the first part of the problem and the decision not to act on the last part of the problem.

SECOND SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #1

Let $t = n - 1$
$E(t) = ae^{\lambda t}$
$a = 500$
$5000 = 500e^{\lambda(1)}$
$\frac{5000}{500} = e^{\lambda}$

$\lambda = \ell n\ 10 = 2.30$
$E(t) = 500e^{2.3t}$

$y = mx + b$
$m = \frac{5000 - 500}{2 - 1} = 4500$
$y = 4500x + b$
$b = 500$
$y = 4500x + 500$
$y = mx + b \longrightarrow y = \frac{m}{2}x^2 + bx + c$
$y = 2250x^2 + 500x + c$
$5000 = 2250 + 500 + c$
$c = 2250$
$Q(x) = 2250x^2 + 500x + 2250$

$n = 4$
$E(t) = E(3) = 500e^{2.3(3)} = 496{,}137$
$Q(x) = Q(4) = 2250(4)^2 + 500(4) + 2250 = 40{,}250$

$\%\ \text{error} = \left(\frac{E(n) - Q(n)}{Q(n)}\right)100$

$\%\ \text{error} = \left(\frac{496{,}137 - 40{,}250}{40{,}250}\right)100$
$\%\ \text{error} = 1{,}133\%$

ANALYSIS FOR SECOND WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

This is an example of a weak response because it is characterized by the following:

Purpose: The candidate demonstrates understanding of the general purpose of the problem but not the knowledge to carry it out correctly. While the candidate demonstrates understanding of the concept of creating an exponential model, the candidate only carries this process out in a memorized standard form. The candidate does not show the flexibility to adapt the procedure to the specified form. The candidate does not create a quadratic model and attempts to use integral calculus to adapt a linear model. This approach does not work. The candidate does carry out the last two charges correctly even though the results are invalidated by errors in the first two charges.

Subject Matter Knowledge: The candidate demonstrates a functional knowledge of techniques for generating standard exponential and linear models from a set of data, although an error is made in calculating the y-intercept for the linear model. The response suggests that the candidate's understanding of mathematical modeling is not deep enough to apply to a variety of function forms such as exponential functions with various bases. The candidate's exponential model does, however, show an understanding of standard exponential modeling. The attempt at a quadratic model reveals several underlying misconceptions. The candidate is missing the basic premise that a quadratic function cannot be determined by less than three points. The candidate's misuse of integration to generate a quadratic function from a linear function shows serious misconceptions in the field of integral calculus. The candidate does, however, correctly demonstrate the use of mathematical models to make predictions as well as the ability to calculate percent error.

Support: The response is clearly and logically laid out. Even though there is no verbal commentary in the response, the candidate's reasoning, though flawed, is easy to follow. There is no need to further document the candidate's decision-making process. Many of the candidate's decisions are incorrect and based on misconceptions (e.g., a quadratic model can be derived from only two data points).

Rationale: The candidate demonstrates a basic understanding of mathematical modeling. However, the response suggests that the candidate's understanding is not deep enough to be transferable to unfamiliar applications. In an attempt to go beyond the candidate's skill level, the candidate tries to use integral calculus. This illuminates not only the candidate's limitations in mathematical modeling, but also some major misconceptions about the use of integral calculus.

FIRST SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #1

$E(n) = ab^n$

$E(1) = ab^1 = 500$ $\qquad E(2) = ab^2 = 5{,}000$

$a = \frac{500}{b} \longrightarrow \left(\frac{500}{b}\right)(b^2) = 5{,}000$

$500b = 5{,}000$

$b = 10$

$ab = 500$

$10a = 500$

$a = 50$

$\boxed{E(n) = 50 \cdot 10^n}$

$Q(n) = an^2 + bn + c$

$Q(1) = a + b + c \Rightarrow$

$Q(2) = 4a + 2b + c \Rightarrow$

$Q(3) = 9a + 3b + c \Rightarrow$

$a + b + c = 500$

$4a + 2b + c = 5{,}000$

$9a + 3b + c = 50{,}000$

$-1(a + b + c) = -1(500)$

$-a - b - c = -500$

$+ 4a + 2b + c = 5{,}000$

$3a + b = 4{,}500$

$b = 4{,}500 - 3a$

$-a - b - c = -500$

$+ 9a + 3b + c = 50{,}000$

$8a + 2b = 49{,}500$

$8a + 2(4{,}500 - 3a) = 49{,}500$

$8a + 9{,}000 - 6a = 49{,}500$

$2a = 40{,}500$

$a = 20{,}250$

$b = 4{,}500 - 3(20{,}250)$

$b = -56{,}250$

$20{,}250 - 56{,}250 + c = 500$

$c = 36{,}500$

$\boxed{Q(n) = 20{,}250n^2 - 56{,}250n + 36{,}500}$

$Q(4) = 20{,}250(16) - 56{,}250(4) + 36{,}500 = 324{,}000 - 225{,}000 + 36{,}500$

$\boxed{= 135{,}500}$

$E(4) = 50 \cdot 10^4 = 50 \cdot 10{,}000$

$\boxed{= 500{,}000}$

Percent Error:

$$\frac{|\text{Estimated} - \text{Actual}|}{\text{Actual}} = \frac{|500{,}000 - 135{,}500|}{135{,}500}$$

$$= \frac{364{,}500}{135{,}500}$$

≈ 2.69 or $\boxed{269\%}$

ANALYSIS FOR FIRST STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

This is an example of a strong response because it is characterized by the following:

Purpose: The candidate fully achieves the purpose of the assignment. The solution is presented in a clear, logical, step-by-step sequence. Each task is carried out correctly and in a manner consistent with the intent of the problem.

Subject Matter Knowledge: The candidate demonstrates a strong knowledge of mathematical modeling, exponential and quadratic functions, solving systems of linear and rational equations, and calculating percent error. The response shows accurate and appropriate application of subject matter knowledge. The derivation of both functions is clearly delineated showing in-depth knowledge of both concepts and procedures.

Support: The candidate misses no steps in the presentation of the solution. In deriving the exponential function, the candidate clearly illustrates the strategy used to determine the values of a and b. In deriving the quadratic function, the three linear equations are clearly delineated followed by their solution using the elimination method. The candidate then shows each step leading to the prediction of year four with each model. The candidate shows the calculations necessary to compute the percent error. No verbal explanation is necessary as each step is fully explained numerically.

Rationale: Each section of the response is first laid out with the general rule and then methodically developed towards the specific solution. This overall organization shows a comprehension of the structure and intent of the problem. It is evident throughout the carefully developed presentation that this candidate thoroughly understands the subject matter.

SECOND SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #1

Exponential Model

$E(n) = ab^n$ (general function form)

$E(1) = ab^1 = 500$ $E(2) = ab^2 = 5000$ (substitute in $n = 1$ and $n = 2$)

$a = \frac{500}{b}$ → $\left(\frac{500}{b}\right)(b^2) = 5000$ (combine results)

$500b = 5000$

$b = 10$

$ab = 500$

$10a = 500$ (substitute b back into first equation)

$a = 50$ (solve for a)

answer: $E(n) = 50 \cdot 10^n$

I substituted values for year number and annual profit into the general exponential function for years one and two. I then solved the resulting system of simultaneous equations to find values for a and b. Plugging in the values for a and b, I derived the exponential model.

Quadratic Model

$Q(n) = an^2 + bn + c$ (general function form)

(substitute in $n = 1$) $Q(1) = a + b + c = 500$

$a = 500 - b - c$ → (substitute)

(substitute in $n = 3$) $Q(3) = 9a + 3b + c = 50{,}000$

$Q(3) = 9(500 - b - c) + 3b + c = 50{,}000$

$4500 - 9b - 9c + 3b + c = 50{,}000$

$-6b - 8c = 45{,}500$ (combine terms)

$-8c = 45{,}500 + 6b$

$c = -5687.5 - \frac{3}{4}b$ (substitute)

$c = -5687.5 - \frac{3}{4}(-56250)$

$c = -5687.5 + 42187.5$

$c = 36{,}500$

(substitute in $n = 2$) $Q(2) = 4a + 2b + c = 5000$

$Q(2) = 4(500 - b - c) + 2b + c = 5000$

$2000 - 4b - 4c + 2b + c = 5000$

$-2b - 3c = 3000$ (combine terms)

$-2b - 3(-5687.5 - \frac{3}{4}b) = 3000$

$-2b + 17062.5 + 2\frac{1}{4}b = 3000$

$\frac{1}{4}b = -14062.5$

$b = -56250$

$a = 500 - b - c = 500 - (-56250) - 36500 = 20250$

answer: $Q(n) = 20{,}250n^2 - 56{,}250n + 36{,}500$

I substituted values for year number and annual profit into the general quadratic function for years one, two, and three. I then solved the resulting system of simultaneous equations to find values for a, b, and c. Plugging in the values for a, b, and c, I derived the quadratic model.

SECOND SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #1 (continued)

Year 4 Predictions

$Q(4) = 16(20{,}250) - 56{,}250(4) + 36{,}500 = 324{,}000 - 22{,}500 + 36{,}500 =$

answer: $Q(4) = 135{,}500$

$E(4) = 50 \cdot 10^4 = 50 \cdot 10{,}000 =$ answer: $E(4) = 500{,}000$

I evaluated my generated quadratic and exponential models for $n = 4$.

Percent Error (divide the difference by the actual value)

$$\frac{|\text{Estimated} - \text{Actual}|}{\text{Actual}} = \frac{|500{,}000 - 135{,}000|}{135{,}000}$$

$$= \frac{364{,}500}{135{,}000}$$

$= 2.69$ or answer: percent error = 269%

I used $E(4)$ as the estimated value and I used $Q(4)$ as the actual value. I then plugged these into the percent error formula to calculate the percent error. The large result shows how dangerous it can be to use a model generated from limited data to make predictions.

ANALYSIS FOR SECOND STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #1

This is an example of a strong response because it is characterized by the following:

Purpose: The candidate carries out each of the four charges accurately. The response shows a logical progression from what is given to what is requested. The purpose of the assignment is fully achieved.

Subject Matter Knowledge: The candidate shows a firm grasp of the process of modeling data with various types of functions, using the resulting models to make predictions, and assessing the percent error of those predictions. The candidate's calculation of the percent error demonstrates a clear understanding of the distinction between theoretical models and actual observed data. The candidate demonstrates an understanding of when and how to use the method of substitution to solve systems of simultaneous linear and rational equations.

Support: The candidate shows each step in each process, leaving no doubt as to the calculations and manipulations involved. Whenever the flow of the solution diverges from a straight vertical progression, the candidate indicates the divergence through the use of arrows. While the candidate often inserts verbal explanations, the response is strong without the text because the mathematical calculations involved are self-explanatory.

Rationale: The solution is clear, accurate, and error free. The presentation is logical and easy to follow. Each part of the problem is solved correctly and the entire solution holds together as an interrelated whole.

FIRST SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #2

$V_b = \frac{4}{3}\pi r^3$

$V_b(d) = \frac{\pi}{6}d^3$

$V_s = \pi r^2 h$

$V_s(r) = V_s(.5) = \pi(2.5)12 = 30\pi^3$

$V_b d = V_s$

$\frac{\pi}{6}d^3 = 30\pi^3$

ANALYSIS FOR FIRST WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

This is an example of a weak response because it is characterized by the following:

Purpose: The purpose is partially achieved. The candidate finds the volume of a single ball and the volume of a piece of stock (with a calculation error) in terms of the diameter. The candidate does not demonstrate an understanding of the relationship between speed of the stock feed and ball production and the respective volumes of the balls and of the stock. The last part of the problem is not attempted.

Subject Matter Knowledge: The candidate uses the formula for the volume of a sphere correctly. The candidate makes an error converting the cylinder volume formula to be a function of diameter as opposed to radius. The fact that these volumes are set equal indicates confusion with the statement in the prompt that "The total volume of the rubber stock fed into the machine is exactly equal to the combined volume of rubber balls manufactured." The candidate ignores the fact that three rubber balls are produced per second. In general, the candidate shows no understanding of the role of rates in the problem.

Support: The candidate skips from $V_b = \frac{4}{3}\pi r^3$ to $V_b(d) = \frac{\pi}{6}d^3$ without showing any computation. There is an error in squaring 0.5 as 2.5, which results in an incorrect response for the volume. The lack of understanding of the speed function formula, i.e., using $\frac{\pi}{6}d^3 = 30\pi^3$ instead of $\frac{\pi}{2}d^3/3\pi$, resulted in the inability to go any further. Even in the few calculations that are shown, there is no verbal or numerical explanation of the candidate's reasoning.

Rationale: The candidate is not able to relate the two volume expressions to speed; therefore the candidate can not derive speed as a function of diameter. Without this function, the candidate can not generate the data points to be graphed. This all had to be inferred since the candidate offered very little evidence of thought processes. The candidate generally shows little, if any, understanding of the problem as a whole.

SECOND SAMPLE WEAK RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #2

In analyzing the rubber ball making machine, the rubber stock fed is 1" in diameter. This stock fed is what the balls are being made out of. This in turn makes the rubber balls with a diameter of one inch.

Volume of rubber ball

Since the ball is a sphere, $V = \frac{4}{3}\pi r^3$

Since the diameter = 1", $r = 0.5$"

Therefore, $V = \frac{4}{3}\pi \cdot 0.5^3 = \frac{4}{3}\pi \cdot 0.125 = 0.523$ cubic inches

Volume of rubber stock

Since the stock is a cylinder, $V = \pi r^2 h$

Since the diameter = 1", $r = 0.5$"

Since the piece of stock is one foot long, $h = 12$"

Therefore, $V = 3.14 \cdot 0.5^2 \cdot 12 = 3.14 \cdot 0.25 \cdot 12 = 9.42$ cubic inches

$S(d)$

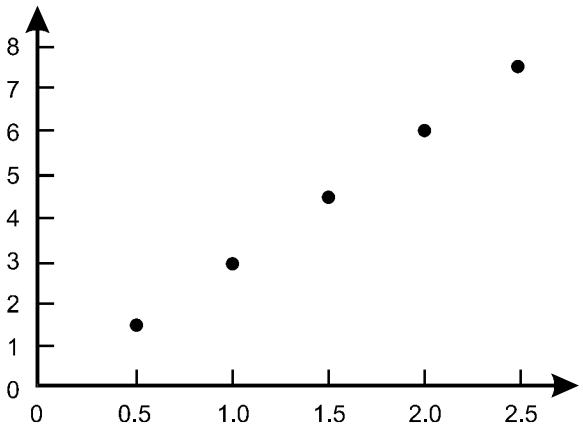
S = speed = 3 balls per second

d = diameter = 1 inch

Therefore, $S(d) = 3 \cdot 1 = 3$

Graph of $S(d)$ for values of d from 0.5" to 2.5"

3(0.5) = 1.5
3(1) = 3
3(1.5) = 4.5
3(2) = 6
3(2.5) = 7.5



ANALYSIS FOR SECOND WEAK RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

This is an example of a weak response because it is characterized by the following:

Purpose: The purpose is partially achieved. The candidate finds the volume of a single ball and the volume of a single piece of stock. In this way, the candidate meets the basic geometry and measurement aspects of the problem. The candidate is not able to make the transition from the concept of static volumes to a volume function. The candidate does not understand the role of speed and related rates in the problem. Finally, the candidate shows rudimentary skills in the graphing of a function although there is some confusion between the meaning of function notation and multiplication.

Subject Matter Knowledge: The candidate demonstrates the ability to determine when and how to apply the two different volume formulas. The candidate is able to adapt the formulas in terms of diameter even though they are given in terms of radius. The candidate shows little, if any, knowledge of functions and function notation. The candidate does not demonstrate the ability to show the relationship between the rate of rubber stock fed into the machine and the rate of rubber ball production. The candidate demonstrates the basic skill of plotting points. The method of generating the data points, however, is limited and does not provide evidence of understanding of functions and function notation. The fact that only the discrete points are plotted demonstrates a lack of understanding of the distinction between continuous and discrete functions.

Support: The candidate lays out the steps of the solution in a clear and organized manner. The additional verbal descriptions of the solution process do not add meaning to the response, as the solution process is evident based on the equations shown. The work is missing labels for the axes of the graph, suggesting lack of attention to detail and possible confusion as to what is actually being graphed.

Rationale: The candidate is able to demonstrate only the most basic skills involved in the problem. Besides the use of two standard volume formulas and the plotting of points, the candidate shows little conceptual understanding of the problem. The candidate misses the very important concepts of functions, related rates, and comparative volumes, the central concepts of the problem.

FIRST SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #2

The formula for volume of a sphere is $V = \frac{4}{3}\pi r^3$

Because $r = \frac{d}{2}$ Then $V_b(d) = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$

$= \frac{4}{3}\pi\left(\frac{d^3}{8}\right)$

$V_b(d) = \frac{\pi}{6}d^3$ in^3 (Volume of 1 ball)

and the volume of 3 balls in the problem

would be $3\left(\frac{\pi}{6}d^3\right) = \frac{\pi}{2}d^3$ in^3

Because 1 linear foot of stock is a cylinder, with diameter 1" and length 12"

Volume = A_{base} × height

$= \pi r^2 \bullet h$

$= \pi\left(\frac{1}{2}\right)^2 \bullet 12 = 3\pi$ in^3

The speed per second to produce the balls is the ratio of the volume of the 3 balls produced compared to the volume of one linear foot of stock used.

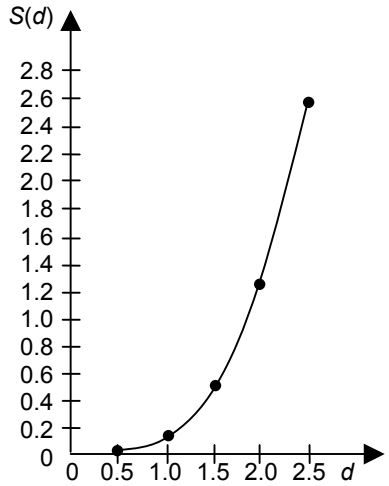
So $S(d) = V_{3\ balls}$ / Volume of 1 linear foot of stock

$S(d) = \frac{\pi}{2}d^3 / V_s$

$= \frac{\pi}{2}d^3 / 3\pi$

$= \frac{d^3}{6}$ feet / sec

d	$S(d)$
0.5	0.02
1.0	0.17
1.5	0.56
2.0	1.33
2.5	2.60



ANALYSIS FOR FIRST STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

This is an example of a strong response because it is characterized by the following:

Purpose: The purpose of the assignment is fully achieved. The volume of a single ball, $V_b(d)$, in cubic inches is found and used to calculate the volume of three balls. The volume of a foot-long piece of stock is found using the formula for the volume of a cylinder. The speed of stock feed required to produce balls of various diameters is expressed as a function of the diameter. The speed function is then graphed.

Subject Matter Knowledge: By correctly converting the formulas to be expressed in terms of diameter, the candidate demonstrates understanding that the formulas for the volume of a sphere and the volume of a cylinder are needed to solve this problem. The explanation, "The speed per second to produce the balls is the ratio of the volume of the 3 balls produced compared to the volume of one linear foot of stock," shows a sophisticated understanding, relating the concepts of volume and speed.

Support: The response provides a detailed step-by-step approach showing formulas (e.g., $V = \frac{4}{3}\pi r^3$), substitution for given data (e.g., $V_b(d) = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$), calculations (e.g., $\frac{4}{3}\pi\left(\frac{d}{2}\right)^3$), and final results (e.g., $V_b(d) = \frac{\pi}{6}d^3$). The response contains verbal explanations of reasoning involved, although it does not substantially contribute to making this a strong response because each step is fully explained numerically. The graph is correctly labeled with an organized supporting chart of data points.

Rationale: Each step is fully explained and justified, which makes the candidate's reasoning process clear. The data table and resulting graph allow for a broad interpretation of the problem beyond producing three balls of a specific size. This demonstrates the candidate's depth of understanding of the underlying concepts.

SECOND SAMPLE STRONG RESPONSE FOR OPEN-RESPONSE ITEM ASSIGNMENT #2

$V_b(r) = \frac{4}{3}\pi r^3$

$r = \frac{d}{2}$

$V_b(d) = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$

$V_b(d) = \frac{4}{3}\pi\frac{d^3}{8}$

$\boxed{V_b(d) = \frac{1}{6}\pi d^3 \text{ in}^3}$

$V_{cyl} = \pi r^2 h$

$h = 12$ in

$r = \frac{1}{2}$ in

$V_s = \pi\left(\frac{1}{2}\right)^2(12)$

$V_s = \pi\left(\frac{1}{4}\right)(12)$

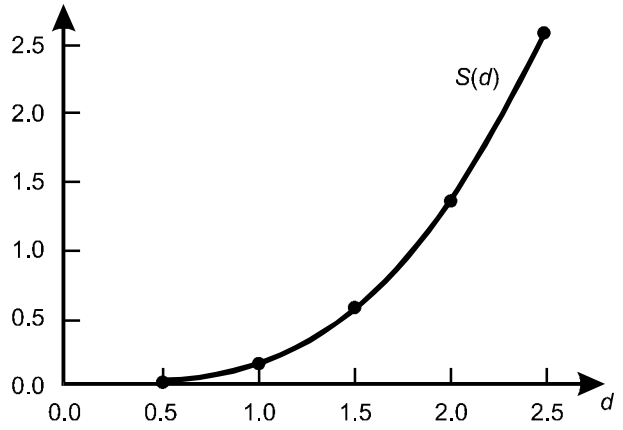
$\boxed{V_s = 3\pi \text{ in}^3}$

$S(d) = \frac{3V_b(d)}{V_s}\ \frac{\text{in}^3/\text{sec}}{\text{in}^3/\text{foot}}$

$S(d) = 3\left(\frac{1}{6}\pi d^3\right)/(3\pi)$

$\boxed{S(d) = \frac{d^3}{6}\frac{\text{feet}}{\text{sec}}}$

$S(0.5) = (0.5)^3/6 \approx 0.021$
$S(1.0) = (1.0)^3/6 \approx 0.167$
$S(1.5) = (1.5)^3/6 \approx 0.563$
$S(2.0) = (2.0)^3/6 \approx 1.333$
$S(2.5) = (2.5)^3/6 \approx 2.604$



ANALYSIS FOR SECOND STRONG RESPONSE TO OPEN-RESPONSE ITEM ASSIGNMENT #2

This is an example of a strong response because it is characterized by the following:

Purpose: The candidate carries out each charge accurately and completely. The volume of a single ball is derived from the formula for the volume of a sphere by substituting $\frac{d}{2}$ for r. The volume of a linear foot of the rubber stock is derived from the formula for the volume of a cylinder by substituting in the dimensions of the foot of stock in inches. The function for the speed of stock feed in feet per second is derived from the quotient of the volume of three balls and the volume of one foot of rubber stock. The speed function is then graphed according to the specifications in the fourth charge.

Subject Matter Knowledge: The candidate shows a sophisticated understanding of volume, including the way a two-to-one ratio between radius and diameter translates to an eight-to-one ratio in the respective volume formulas. The candidate is also able to make the abstract conceptual leap from a one-foot length of stock to a cylinder of height equal to 12 inches. The candidate is then able to formulate a response to the third charge by relating the answers to the first two charges. This demonstrates a high-level understanding of related rates and dimensional analysis. Finally, the candidate is able to generate a set of data points from the speed function and use them to create a graph. The data points are listed in a manner that shows knowledge of rounding. The points are accurately located between tick marks showing good number sense. The points are then connected with a smooth curve showing a qualitative understanding of cubic functions.

Support: The response contains no verbal explanations and yet each step is perfectly clear and well supported mathematically. The presentation is clearly detailed using indentation for secondary substitutions and boxes for final results, which makes the entire process clear. The candidate details the development of each equation in a vertical progression that is easily followed. By including the units in the first step of the third charge, the candidate uses dimensional analysis to justify the approach. In the fourth charge, the candidate shows the substitution of each value for d and how it is used to generate each value of $S(d)$.

Rationale: The solution is clear and succinct while containing enough information to justify each step. The presentation is accurate, appropriate, and economical with no extraneous information. The candidate demonstrates a mastery of all of the content involved in the problem, including derivation of formulas and functions, spatial reasoning, related rates, dimensional analysis, algebraic manipulation, and creating graphs.

PRACTICE TEST SCORE CALCULATION

The practice test score calculation is provided so that you may better gauge your performance and degree of readiness to take an MTEL test at an operational administration. Although the results of this practice test may be used as one indicator of potential strengths and weaknesses in your knowledge of the content on the official test, it is not possible to predict precisely how you might score on an official MTEL test.

The Sample Responses and Analyses for the open-response items may help you determine whether your responses are more similar to the strong or weak samples. The Scoring Rubric can also assist in estimating a score for your open responses. You may also wish to ask a mentor or teacher to help evaluate your responses to the open-response questions prior to calculating your total estimated score.

How to Calculate Your Practice Test Score

Review the directions in the sample below and then use the blank practice test score calculation worksheet on the following page to calculate your estimated score.

SAMPLE

Multiple-Choice Section

Enter the total number of multiple-choice questions you answered correctly:	63	
Use Table 1 below to convert that number to the score and write your score in **Box A**:		**A:** 192

Open-Response Section

Enter the number of points (1 to 4) for your first open-response question:	2	
Enter the number of points (1 to 4) for your second open-response question:	4	
	======	
Add those two numbers (Number of open-response question points):	6	
Use Table 2 below to convert that number to the score and write your score in **Box B**:		**B:** 52

Total Practice Test Score (Estimated MTEL Score)

Add the numbers in **Boxes A and B** for an estimate of your MTEL score:	**A + B =** 244

Practice Test Score Calculation Worksheet: Mathematics

Table 1:

Number of Multiple-Choice Questions Correct	Estimated MTEL Score	Number of Multiple-Choice Questions Correct	Estimated MTEL Score
0 to 25	140	61 to 65	192
26 to 30	147	66 to 70	198
31 to 35	153	71 to 75	205
36 to 40	160	76 to 80	211
41 to 45	166	81 to 85	218
46 to 50	173	86 to 90	224
51 to 55	179	91 to 95	231
56 to 60	186	96 to 100	237

Table 2:

Number of Open-Response Question Points	Estimated MTEL Score
2	36
3	40
4	44
5	48
6	52
7	56
8	60

Print the form below to calculate your estimated practice test score.

Multiple-Choice Section

Enter the total number of multiple-choice questions you answered correctly:

Use Table 1 above to convert that number to the score and write your score in **Box A**: **A:** ____

Open-Response Section

Enter the number of points (1 to 4) for your first open-response question:

Enter the number of points (1 to 4) for your second open-response question:

======

Add those two numbers (Number of open-response question points):

Use Table 2 above to convert that number to the score and write your score in **Box B**: **B:** ____

Total Practice Test Score (Estimated MTEL Score)

Add the numbers in **Boxes A and B** for an estimate of your MTEL score: **A + B =** ____