Massachusetts Tests for Educator Licensure™

FIELD 09: MATHEMATICS
TEST OBJECTIVES

Subarea	Multiple-Choice	Range of Objectives	Approximate Test Weighting
I.	Number Sense and Operations	01–03	12%
II.	Patterns, Relations, and Algebra	04–10	23%
III.	Geometry and Measurement	11–15	19%
IV.	Data Analysis, Statistics, and Probability	16–18	10%
V.	Trigonometry, Calculus, and Discrete Mathematics	19–23	16%
			80%
	Open-Response		
VI.	Integration of Knowledge and Understanding	24	**20%**

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Test Objectives
Field 09: Mathematics

SUBAREAS:

NUMBER SENSE AND OPERATIONS
PATTERNS, RELATIONS, AND ALGEBRA
GEOMETRY AND MEASUREMENT
DATA ANALYSIS, STATISTICS, AND PROBABILITY
TRIGONOMETRY, CALCULUS, AND DISCRETE MATHEMATICS
INTEGRATION OF KNOWLEDGE AND UNDERSTANDING

NUMBER SENSE AND OPERATIONS [12%]

0001 Understand the structure of numeration systems and solve problems using integers, fractions, decimals, percents, ratios, and proportions.

For example: place value; order relationships; relationships between operations (e.g., division as the inverse of multiplication); multiple representations of numbers and of number operations (e.g., area models for multiplication); absolute value; signed numbers; computational algorithms; problems involving integers, fractions, decimals, percents, ratios, and proportions; the use of estimation to judge the reasonableness of solutions to problems; the origins and development of standard computational algorithms; and properties of early numeration systems (e.g., Mayan, Mesopotamian, Egyptian).

0002 Understand the properties of real and complex numbers and the real and complex number systems.

For example: rational and irrational numbers; multiple representations of complex numbers (e.g., vector, trigonometric, exponential); properties (e.g., closure, distributive, associative) of the real and complex number systems and their subsets; operations on complex numbers; the laws of exponents; calculating roots and powers of real and complex numbers; scientific notation; using number properties to prove theorems; and problems involving real and complex numbers and their operations.

0003 Understand the principles of number theory.

For example: number factors and divisibility; prime and composite numbers; prime factorization; Euclid's algorithm; congruence classes and modular arithmetic; Mersenne primes and perfect numbers; statement of Fermat's Last Theorem; and the fundamental theorem of arithmetic.

Field 09: Mathematics
Test Objectives

PATTERNS, RELATIONS, AND ALGEBRA [23%]

0004 Understand and use patterns to model and solve problems.

For example: conjectures about patterns presented in numeric, geometric, or tabular form; representation of patterns using symbolic notation; identification of patterns of change created by functions (e.g., linear, quadratic, exponential); iterative and recursive functional relationships (e.g., Fibonacci numbers); Pascal's triangle and the binomial theorem; and using finite and infinite sequences and series (e.g., arithmetic, geometric) to model and solve problems.

0005 Understand the properties of functions and relations.

For example: the difference between relations and functions; multiple ways of representing functions (e.g., tabular, graphic, symbolic, verbal); properties of functions and relations (e.g., domain, range, continuity); piecewise-defined functions; addition, subtraction, and composition of functions; inverse functions; and graphs of functions and their transformations [e.g., the relationships among $f(x)$, $f(x + k)$, $f(x) + k$, $kf(x)$].

0006 Understand the properties and applications of linear relations and functions.

For example: the relationship between linear models and rate of change; direct variation; graphs of linear equations; slopes and intercepts of lines; finding an equation for a line; algebraic, numeric, and graphical methods of solving systems of linear equations and inequalities; expressions involving absolute value; and using a variety of methods to model and solve problems involving linear functions and systems.

0007 Understand the properties and applications of linear and abstract algebra.

For example: properties of matrices and determinants; representing and solving linear systems using matrices; geometric and algebraic properties of vectors; properties of vector spaces (e.g., basis, dimension); the matrix representing a linear transformation; and the definitions and properties of groups, rings, and fields.

0008 Understand the properties and applications of quadratic relations and functions.

For example: manipulation and simplification of quadratic expressions; methods of solving quadratic equations and inequalities (e.g., factoring, completing the square, quadratic formula, graphing); real and complex roots of quadratic equations; graphs of quadratic functions; relationship between the graphic and symbolic representations of quadratic functions; quadratic maximum and minimum problems; and modeling and solving problems using quadratic relations, functions, and systems.

0009 Understand the properties and applications of polynomial, radical, rational, and absolute value functions and relations.

For example: inverse and joint variation problems; zeros of polynomial functions; manipulating and simplifying polynomial and rational expressions; horizontal and vertical asymptotes; and properties and graphs of and modeling and solving problems involving polynomial, radical, rational, absolute value, and step functions.

0010 Understand the properties and applications of exponential and logarithmic functions and relations.

For example: simplifying exponential and logarithmic expressions; properties and graphs of exponential and logarithmic functions; problems involving exponential growth, decay, and compound interest; applications of logarithmic functions (e.g., decibel scale, Richter scale); and using the inverse relationship between exponential and logarithmic functions to solve problems.

GEOMETRY AND MEASUREMENT [19%]

0011 Understand the principles, concepts, and procedures related to measurement.

For example: unit conversions within and among measurement systems; dimensional analysis; problems involving length, area, volume, mass, capacity, density, time, temperature, angles, and rates of change; degree and radian measure; indirect measurement; the effect of changing linear dimensions on measures of length, area, or volume; and the effects of measurement error and rounding on computed quantities (e.g., area, density, speed).

0012 Understand the axiomatic structure of Euclidean geometry.

For example: the nature of axiomatic systems; undefined terms, postulates, and theorems; relationships among points, lines, rays, angles, and planes; axioms of algebra (e.g., addition postulate), distance and angle measure postulates; special pairs of angles (e.g., supplementary, vertical); properties of parallel and perpendicular lines and planes; triangle congruence conditions; similar triangles; Pythagorean theorem; segments and angles associated with circles; and the origins and development of geometry in different cultures (e.g., Greek, Hindu, Chinese).

0013 Prove theorems within the axiomatic structure of Euclidean geometry.

For example: direct and indirect methods of proof; properties of parallel and perpendicular lines as they relate to polygons and circles; congruent triangles; properties of special triangles; characteristics of parallelograms and other quadrilaterals; similar triangles and other polygons; geometric constructions; and theorems associated with the properties of circles.

0014 Apply Euclidean geometry to solve problems involving two- and three-dimensional objects.

For example: special right triangle relationships; arcs, angles, and segments associated with polygons and circles; properties of three-dimensional figures (e.g., prisms, pyramids, cylinders, cones); perspective drawings and projections; cross sections (including conic sections) and nets; generating three-dimensional figures from two-dimensional shapes; and using two- and three-dimensional models to solve problems.

0015 Understand the principles and properties of coordinate and transformational geometry and characteristics of non-Euclidean geometries.

For example: rectangular and polar coordinates; representation of geometric figures (e.g., lines, triangles, circles) in the coordinate plane; three-dimensional coordinate systems; using concepts of distance, midpoint, slope, and parallel and perpendicular lines to classify and analyze figures (e.g., parallelograms, conic sections); characteristics of dilations, translations, rotations, reflections, and glide-reflections; types of symmetry; transformations in the coordinate plane; and axioms and features of non-Euclidean geometries (e.g., hyperbolic, elliptic).

DATA ANALYSIS, STATISTICS, AND PROBABILITY [10%]

0016 Understand the principles and concepts of descriptive statistics and their application to the problem-solving process.

For example: choosing, constructing, and interpreting appropriate tables, charts, and graphs (e.g., line plots, stem-and-leaf plots, box plots, histograms, circle graphs); measures of central tendency (e.g., mean, median, mode) and dispersion (e.g., range, standard deviation, interquartile range); frequency distributions; percentile scores; and the effects of data transformations on measures of central tendency and variability.

0017 Understand the methods used in collecting and analyzing data.

For example: evaluating real-world situations to determine appropriate sampling techniques and methods for gathering data (e.g., random sampling, avoidance of bias); designing statistical experiments; making appropriate inferences about a population from sample statistics; effects of sample size; interpreting correlation; and problems involving regression models and curve fitting.

0018 Understand the fundamental principles of probability.

For example: probabilities for simple and compound events (e.g., dependent, independent, and mutually exclusive events, conditional probability); the use of simulations to explore probability; probability as a ratio of two areas; and using random variables and probability distributions (e.g., uniform, normal, binomial) to solve problems.

TRIGONOMETRY, CALCULUS, AND DISCRETE MATHEMATICS [16%]

0019 Understand the properties of trigonometric functions and identities.

For example: degree and radian measure; right triangle trigonometry; the laws of sines and cosines; the relationship between the unit circle and trigonometric functions; graphs and properties (e.g., amplitude, period, phase shift) of trigonometric functions and their inverses; trigonometric identities; solving trigonometric equations; and using trigonometric functions to model periodic phenomena.

0020 Understand the concepts of limit, continuity, and rate of change.

For example: limits of algebraic functions and of infinite sequences and series (including the geometric series); continuous and discontinuous functions; the relationship between the secant line and the average rate of change of a function; and solving problems involving average rates of change.

0021 Understand differential calculus.

For example: the slope of the line tangent to a curve; definition and properties of the derivative; differentiability; techniques of differentiation (e.g., product rule, chain rule); the derivative of algebraic and transcendental functions; analyzing the graph of a function; using differentiation to solve problems (e.g., velocity, acceleration, optimization, related rates); verifying that a given function is a solution of a differential equation; and the development of differential calculus.

0022 Understand integral calculus.

For example: algebraic and geometric approximations of the area under a curve; the integral as the limit of a Riemann sum; the fundamental theorem of calculus; techniques of integration; applications of integration (e.g., area, work, volume, arc length, displacement, velocity); and solving differential equations by separation of variables.

0023 Understand the principles of discrete/finite mathematics.

For example: properties of sets; counting techniques (e.g., permutations, combinations); finite differences; the mathematics of finance (e.g., compound interest, annuities, amortization); recursive patterns and relations; iteration; properties of algorithms; linear programming in two variables; properties of matrices; and characteristics and applications of finite graphs and trees.

INTEGRATION OF KNOWLEDGE AND UNDERSTANDING [20%]

In addition to answering multiple-choice items, candidates will prepare written responses to questions addressing content from the preceding objectives, which are summarized in the objective and descriptive statement below.

0024 Prepare an organized, developed analysis emphasizing problem solving, communicating, reasoning, making connections, and/or using representations on topics related to one or more of the following: number sense and operations; patterns, relations, and algebra; geometry and measurement; data analysis, statistics, and probability; trigonometry, calculus, and discrete mathematics.

For example: presenting a detailed solution to a problem involving one or more of the following: place value, number base, and the structure and operations of number systems; properties, attributes, representations, and applications of families of functions; modeling real-world problems with functions; applications of plane and three-dimensional geometry; Euclidean geometry and proof; connections between algebra and geometry; and design, analysis, presentation, and interpretation of statistical surveys.