Massachusetts Tests for Educator Licensure®

TEST INFORMATION BOOKLET

53 Elementary Mathematics

MA-SG-FLD053-03

Massachusetts Department of Education

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(Field 53)

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Test Overview Chart: Elementary Mathematics (53)

Subareas	Approximate Number of Multiple-Choice Items	Number of Open-Response Items
I. Number Sense, Operations, and Data Analysis	37-39	
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The Elementary Mathematics test is designed to assess the candidate's knowledge of the subject matter required for the Massachusetts Elementary Mathematics license. This subject matter knowledge is delineated in the Massachusetts Department of Education's *Regulations for Educator Licensure and Preparation Program Approval* (7/2001), 603 CMR 7.06 "Subject Matter Knowledge Requirements for Teachers."

The Elementary Mathematics test assesses the candidate's proficiency and depth of understanding of the subject at the level required for a baccalaureate major, according to Massachusetts standards. Candidates are typically nearing completion of or have completed their undergraduate work when they take the test.

The multiple-choice items on the test cover the subareas as indicated in the chart above. The open-response items may relate to topics covered in any of the subareas and will typically require breadth of understanding of the elementary mathematics field and the ability to relate concepts from different aspects of the field. Responses to the open-response items are expected to be appropriate and accurate in the application of subject matter knowledge, to provide high-quality and relevant supporting evidence, and to demonstrate a soundness of argument and understanding of the elementary mathematics field.

Massachusetts Tests for Educator Licensure™

FIELD 53: ELEMENTARY MATHEMATICS
TEST OBJECTIVES

Subarea	Multiple-Choice	Range of Objectives	Approximate Test Weighting
I.	Number Sense, Operations, and Data Analysis	01–06	30%
II.	Patterns, Relations, and Algebra	07–11	25%
III.	Geometry and Measurement	12–15	25%
			80%
	Open-Response		
IV.	Integration of Knowledge and Understanding	16	**20%**

SUBAREAS:

NUMBER SENSE, OPERATIONS, AND DATA ANALYSIS
PATTERNS, RELATIONS, AND ALGEBRA
GEOMETRY AND MEASUREMENT
INTEGRATION OF KNOWLEDGE AND UNDERSTANDING

NUMBER SENSE, OPERATIONS, AND DATA ANALYSIS [30%]

0001 Understand number theory, the structure of numeration systems, and the properties of the real number system.

For example: place value (including base ten and other bases); order relations; the relationships between operations (e.g., multiplication as repeated additions); factors and divisibility; prime and composite numbers; prime factorization; rational and irrational numbers; properties (e.g., closure, distributive, associative) of the real number system; operations and their inverses; the real number line; roots and powers; the laws of exponents; scientific notation; and proof of theorems using number properties (e.g., the product of two even numbers is even).

0002 Understand principles and operations related to integers.

For example: order of operations; identity and inverse elements; associative, commutative, and distributive properties; absolute value; operations with signed numbers; multiple representations of number operations (e.g., area models); rounding and estimation; standard algorithms for addition, subtraction, multiplication, and division; number operations and their inverses; the origins and development of standard computational algorithms; the relationship between properties of integers and the development of algebraic concepts; and problems involving integers.

0003 Understand principles and operations related to fractions.

For example: multiple representations of fractions (e.g., area and set models); multiple representations of operations involving fractions; equivalent fractions; addition, subtraction, multiplication, and division of fractions; comparison, ordering, and estimation of fractions; placement on a number line; simplification of fractions; mixed numbers and improper fractions; the relationship between properties of fractions and the development of algebraic concepts; and problems involving fractions.

0004 Understand principles and operations related to decimals and percents.

For example: multiple representations of decimals; addition, subtraction, multiplication, and division of decimals; ordering decimals and placement on a number line; terminating and repeating decimals; rounding and estimation; conversion among decimals, percents, and fractions; the relationship between properties of decimals and the development of algebraic concepts; and problems involving decimals and percents.

0005 Understand descriptive statistics and the methods used in collecting, organizing, reporting, and analyzing data.

For example: construction and interpretation of tables, charts, and graphs (e.g., line plots, stem-and-leaf plots, box plots, scatter plots); measures of central tendency (e.g., mean, median, mode) and dispersion (e.g., range, standard deviation); frequency distributions; percentile scores; evaluation of real-world situations to determine appropriate sampling techniques and methods for gathering and organizing data; line of best fit; and appropriate inferences, interpolations, and extrapolations from a set of data.

0006 Understand the fundamental principles of probability.

For example: sample space for a probabilistic situation (e.g., events or outcomes); counting principles (e.g., permutations and combinations); computation of experimental and theoretical probabilities for simple and compound events; simulations that explore real-world situations; connections between geometry and probability (e.g., probability as a ratio of two areas); and probability models that represent real-world phenomena.

PATTERNS, RELATIONS, AND ALGEBRA [25%]

0007 Understand patterns and the properties of functions and relations.

For example: conjectures about patterns presented in numeric, geometric, or tabular form; representation of patterns and relations using symbolic notation; identification of patterns of change created by functions (e.g., linear, quadratic, exponential); iterative and recursive patterns and relationships; various types of relations including functions; generation and interpretation of graphs that model real-world situations; domain and range; multiple representations of relations (e.g., tabular, graphic, verbal, symbolic); and properties of functions and relations.

0008 Understand how to manipulate and simplify algebraic expressions and translate problems into algebraic expressions, equations, and inequalities.

For example: the nature of a variable; evaluating algebraic expressions for a given value of a variable; the relationship between standard computational algorithms and algebraic processes (e.g., integer division and polynomial division); expressing direct and inverse relationships algebraically; expressing one variable in terms of another; manipulating and simplifying algebraic expressions (e.g., factoring, applying the properties of exponents); solving equations and inequalities; and modeling situations with algebraic expressions.

0009 Understand properties and applications of linear relations and functions.

For example: the relationship between linear models and rate of change; direct variation; graphs of linear equations; slope and intercepts of lines; finding an equation for a line; solving systems of linear equations and inequalities (e.g., graphing, substitution); and modeling and solving problems using linear functions and systems.

0010 Understand properties and applications of ratios and proportions.

For example: relative and absolute relationships; direct variation and proportion; writing and solving proportions; percents and proportions; and solving problems involving ratios and proportions (e.g., mixtures, rates, scale factors, scale drawings).

0011 Understand properties and applications of quadratic, exponential, polynomial, and rational functions and relations.

For example: methods of solving quadratic equations (e.g., factoring, completing the square, quadratic formula, graphing); nature of roots of quadratic equations (real and imaginary); graphs of quadratic functions; modeling and solving problems using quadratic relations, functions, and systems (e.g., maximum and minimum problems); properties and graphs of exponential functions; solving problems involving exponential growth (e.g., population growth, compound interest) and decay (e.g., half-life); inverse variation; and modeling problems with rational functions.

GEOMETRY AND MEASUREMENT [25%]

0012 Understand principles, concepts, and procedures related to measurement.

For example: appropriate units of measurement; unit conversions within and among measurement systems; problems involving length, perimeter, area, volume, mass, capacity, density, time, temperature, angles, and rates of change; problems involving ratios and proportions, similar plane figures, and indirect measurement; effect of changing linear dimensions on measures of perimeter, area, and/or volume; and effects of measurement error and rounding on computed quantities (e.g., area, density, speed).

0013 Understand the principles of Euclidean geometry and use them to prove theorems.

For example: the nature of axiomatic systems; undefined terms and postulates of Euclidean geometry; relationships among points, lines, angles, and planes; methods for proving triangles congruent; properties of similar triangles; justification of geometric constructions; evaluating the validity of conjectures; and proof of theorems within the axiomatic structure of Euclidean geometry.

0014 Apply Euclidean geometry to analyze the characteristics and properties of two- and three-dimensional shapes.

For example: justification of properties and relationships among triangles, quadrilaterals, and other polygons through deduction; identification of plane figures given characteristics of sides, angles, and diagonals; Pythagorean theorem; special right triangle relationships; arcs, angles, and segments associated with circles; area of composite shapes; surface area and volume of common three-dimensional figures; cross sections and nets; and models and problems involving two- and three-dimensional figures and shapes.

0015 Understand the principles and properties of coordinate and transformational geometry and use them to prove theorems.

For example: representation of polygons in the coordinate plane; concepts of distance, midpoint, slope, and parallel and perpendicular lines to classify and analyze figures (e.g., parallelograms); characteristics of dilations, translations, rotations, reflections, and glide-reflections; types of symmetry; properties of tessellations; transformations in the coordinate plane; and proofs of theorem and solutions of problems through coordinate and transformational geometry.

INTEGRATION OF KNOWLEDGE AND UNDERSTANDING [20%]

In addition to answering multiple-choice items, candidates will prepare written responses to questions addressing content from the preceding objectives, which are summarized in the objective and descriptive statement below.

0016 Prepare an organized, developed analysis on a topic related to one or more of the following: number sense, operations, and data analysis; patterns, relations, and algebra; and geometry and measurement.

For example: present a detailed solution to a problem involving one or more of the following: place value, number bases, and the structure and operations of number systems; evaluation of methods for gathering, organizing, and interpreting data; the relationship between standard computational algorithms and algebraic processes; application of ratios and proportions in a variety of situations; finding probabilities; properties, attributes, and representations of algebraic functions; problems involving rates of change; and applications of plane and three-dimensional geometry.

1. Simplify the expression $\frac{\sqrt[3]{x^4} \cdot \sqrt[4]{y^3}}{\sqrt[3]{x} \cdot \sqrt{y^3}}$.

   A. $x^4 y^{\frac{1}{2}}$

   B. $xy^{-\frac{9}{4}}$

   C. $x^{-\frac{9}{4}} y$

   D. $xy^{-\frac{3}{4}}$

2. **Use the algorithms below to answer the question that follows.**

48	48
× 17	× 17
336	56
48	280
816	80
	400
	816
standard multiplication algorithm	expanded multiplication algorithm

   A significant difference between the standard multiplication algorithm and the expanded multiplication algorithm is that the expanded multiplication algorithm:

   A. can be used with both positive and negative integers.

   B. emphasizes the use of the distributive property.

   C. minimizes the need to memorize multiplication facts.

   D. makes knowledge of place value unnecessary.

3. Simplify the expression below.

$$\frac{\frac{5}{8}+\frac{3}{4}}{\frac{7}{8}-\frac{3}{5}}$$

A. $\frac{1}{2}$

B. $2\frac{1}{2}$

C. 4

D. 5

4. A student rolls two dice. One die has six faces, each face marked with 1, 2, 3, 4, 5, or 6 spots, and the other die has eight faces, each marked with 1, 2, 3, 4, 5, 6, 7, or 8 spots. How many outcomes are in the sample space for this experiment?

A. 14

B. 28

C. 48

D. 64

5. **Use the information below to answer the question that follows.**

A particular function is defined as follows: Take any integer greater than zero. If the integer is even, multiply it by 2. If the integer is odd, add 1 and then multiply the result by 2.

What is the domain of the function?

A. the number 2

B. the set of even integers

C. the set of odd integers

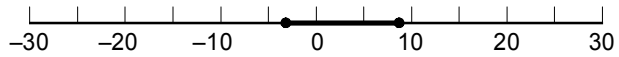
D. the set of positive integers

6. **Use the inequality below to answer the question that follows.**

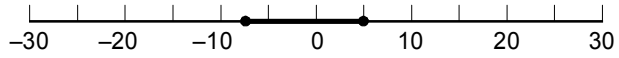
$$-6 \leq \frac{x}{3} - 3 \leq 5$$

Which of the following best represents the solution of the inequality above?

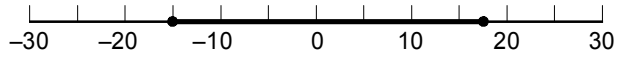
A.



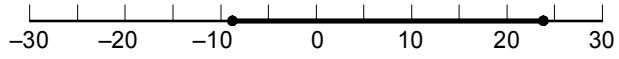
B.



C.



D.



7. A scientist determines that the amount of mold growing in a laboratory culture triples every 4 days. If there is originally 0.5 grams of mold in the culture, which of the following equations best models the amount of mold, M, at a given time t?

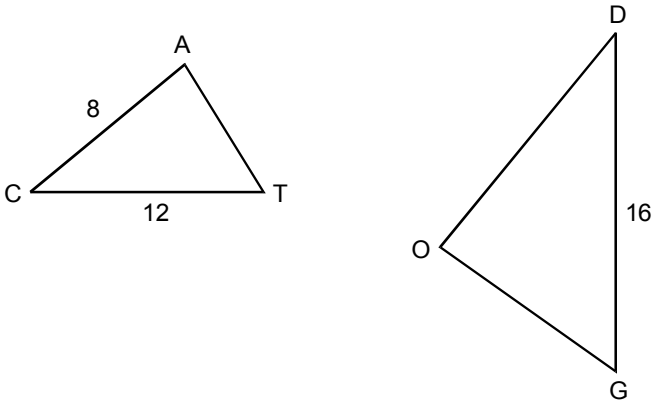
   A. $M = 0.5(3)^{0.25t}$

   B. $M = 0.5(3)^{4t}$

   C. $M = 3t + 0.5$

   D. $M = t^3 + 0.5$

8. **Use the triangles below to answer the question that follows.**



If $\triangle CAT \sim \triangle DOG$, what is the measure of $\overline{DO}$?

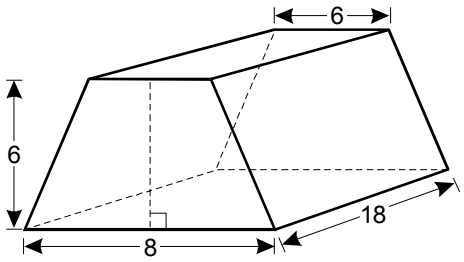
   A. $9\frac{1}{3}$

   B. $10\frac{2}{3}$

   C. 24

   D. 32

9. **Use the diagram below to answer the question that follows.**



The diagram shows a right trapezoidal prism. What is the area of a horizontal cross section of the prism, 3 units above and parallel to the base of the prism?

A. 42

B. 108

C. 126

D. 144

10. An equilateral triangle is constructed on a set of coordinate axes. If one vertex is at the origin and another is at the point (a, 0), which of the following would be the y-coordinate of the third vertex of the triangle?

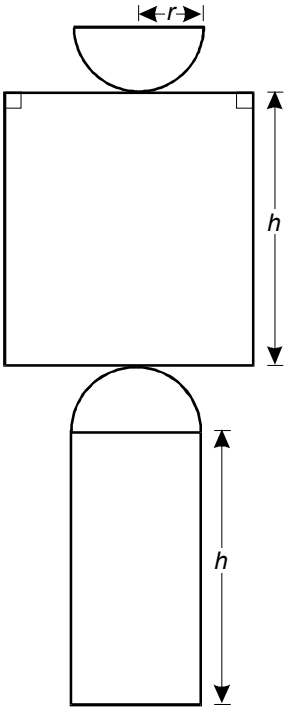
A. $\frac{\sqrt{3}a}{2}$

B. $\frac{3a}{4}$

C. $\frac{\sqrt{3}a}{4}$

D. $\frac{3a}{2}$

11. **Use the diagram below to complete the exercise that follows.**



A "net" is a two-dimensional diagram that can be cut along its edges and folded into a three-dimensional figure. Use your knowledge of measurement and geometry to analyze the characteristics of the three-dimensional figure that the net above represents. In your response, you should:

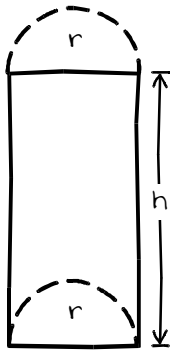
- draw a sketch of the figure with the dimensions labeled as in the diagram, and identify the figure;
- using the dimensions as labeled in the diagram, derive an equation for the volume of the figure; and
- using the dimensions as labeled in the diagram, derive an equation for the surface area of the figure.

Answer Key and Sample Response: Elementary Mathematics (53)

Question Number	Correct Response	Test Objective
1.	**D**	Understand number theory, the structure of numeration systems, and the properties of the real number system.
2.	**B**	Understand principles and operations related to integers.
3.	**D**	Understand principles and operations related to fractions.
4.	**C**	Understand the fundamental principles of probability.
5.	**D**	Understand patterns and properties of functions and relations.
6.	**D**	Understand properties and applications of linear relations and functions.
7.	**A**	Understand properties and applications of quadratic, exponential, polynomial, and rational functions and relations.
8.	**B**	Understand the principles of Euclidean geometry and use them to prove theorems.
9.	**C**	Apply Euclidean geometry to analyze the characteristics and properties of two- and three-dimensional shapes.
10.	**A**	Understand the principles and properties of coordinate and transformational geometry and use them to prove theorems.

The sample response below reflects a strong knowledge and understanding of the subject matter.

1)



This is 1/2 of a half right circular cylinder.

2) A circle has the area of πr^2. There are 2 half circles. The height of the cylinder excluding the 2 half circles is represented by h. However, there is only 1/2 of a cylinder.

Volume equals $1/2\pi r^2 h$

V = volume

π = 3.14

r = radius

h = height

3) As a result of analyzing the above: surface area = the areas of the 2 bases ($2 \times 1/2\pi r^2$) AND the curved surface of the cylinder ($1/2 \times 2\pi rh$) AND the area of the rectangular surface ($2rh$).

Hence the equation for the surface area = $\pi r^2 + \pi rh + 2rh$.