Massachusetts Tests for Educator Licensure®

TEST INFORMATION BOOKLET

51 Middle School Mathematics/Science

MA-SG-FLD051-03

Massachusetts Department of Education

Table of Contents

Middle School Mathematics/Science (Field 51)

Test Overview Chart

Test Objectives

Sample Test Items

Answer Key and Sample Responses

Test Overview Chart: Middle School Mathematics/Science (51)

Subareas	Approximate Number of Multiple-Choice Items	Number of Open-Response Items
I. Number Sense and Operations	9–11	
II. Patterns, Relations, and Algebra	14–16	
III. Geometry and Measurement	10–12	
IV. Data Analysis, Statistics, and Probability	7–9	
V. Trigonometry, Calculus, and Discrete Mathematics	5–7	
VI. History, Philosophy, and Methodology of Science	9–11	
VII. Chemistry	9–11	
VIII. Physics	9–11	
IX. Biology	9–11	
X. Earth and Space Science	9–11	
XI. Integration of Knowledge and Understanding of Mathematics		1
XII. Integration of Knowledge and Understanding of Science		1

The Middle School Mathematics/Science test is designed to assess the candidate's knowledge of the subject matter required for the Massachusetts Middle School Mathematics/Science license. This subject matter knowledge is delineated in the Massachusetts Department of Education's *Regulations for Educator Licensure and Preparation Program Approval* (7/2001), 603 CMR 7.06, "Subject Matter Knowledge Requirements for Teachers."

The Middle School Mathematics/Science test assesses the candidate's proficiency and depth of understanding of the subject at the level required for a baccalaureate major, according to Massachusetts standards. Candidates are typically nearing completion of or have completed their undergraduate work when they take the test.

The multiple-choice items on the test cover the subareas as indicated in the chart above. The open-response items may relate to topics covered in any of the subareas and will typically require breadth of understanding of the middle school mathematics/science field and the ability to relate concepts from different aspects of the field. Responses to the open-response items are expected to be appropriate and accurate in the application of subject knowledge, to provide high-quality and relevant supporting evidence, and to demonstrate a soundness of argument and understanding of the middle school mathematics/science field.

Massachusetts Tests for Educator Licensure (MTEL™)

FIELD 51: MIDDLE SCHOOL MATHEMATICS/SCIENCE TEST OBJECTIVES

Subarea	Multiple-Choice	Range of Objectives	Approximate Test Weighting
I.	Number Sense and Operations	01–04	8%
II.	Patterns, Relations, and Algebra	05–10	12%
III.	Geometry and Measurement	11–15	9%
IV.	Data Analysis, Statistics, and Probability	16–17	6%
V.	Trigonometry, Calculus, and Discrete Mathematics	18–20	5%
VI.	History, Philosophy, and Methodology of Science	21–24	8%
VII.	Chemistry	25–28	8%
VIII.	Physics	29–32	8%
IX.	Biology	33–36	8%
X.	Earth and Space Science	37–40	8%
			80%
	Open-Response		
XI.	Integration of Knowledge and Understanding of Mathematics	41	**10%**
XII.	Integration of Knowledge and Understanding of Science	42	**10%**

SUBAREAS:

NUMBER SENSE AND OPERATIONS
PATTERNS, RELATIONS, AND ALGEBRA
GEOMETRY AND MEASUREMENT
DATA ANALYSIS, STATISTICS, AND PROBABILITY
TRIGONOMETRY, CALCULUS, AND DISCRETE MATHEMATICS
HISTORY, PHILOSOPHY, AND METHODOLOGY OF SCIENCE
CHEMISTRY
PHYSICS
BIOLOGY
EARTH AND SPACE SCIENCE
INTEGRATION OF KNOWLEDGE AND UNDERSTANDING OF MATHEMATICS
INTEGRATION OF KNOWLEDGE AND UNDERSTANDING OF SCIENCE

NUMBER SENSE AND OPERATIONS [8%]

0001 Understand the structure of numeration systems and multiple representations of numbers.

For example: place value; number bases (e.g., base 2, base 10); order relations; relationships between operations (e.g., multiplication as repeated additions); number factors and divisibility; prime and composite numbers; prime factorization; multiple representations of numbers (e.g., physical models, diagrams, numerals); and properties of early numeration systems (e.g., Mayan, Mesopotamian, Egyptian).

0002 Understand principles and operations related to integers, fractions, decimals, percents, ratios, and proportions.

For example: order of operations; identity and inverse elements; associative, commutative, and distributive properties; absolute value; operations with signed numbers; multiple representations (e.g., area models for multiplication) of number operations; analyzing standard algorithms for addition, subtraction, multiplication, and division of integers and rational numbers; number operations and their inverses; and the origins and development of standard computational algorithms.

0003 Understand and solve problems involving integers, fractions, decimals, percents, ratios, and proportions.

For example: solving a variety of problems involving integers, fractions, decimals, percents (including percent increase and decrease), ratios, proportions, and average rate of change; and using estimation to judge the reasonableness of solutions to problems.

0004 Understand the properties of real numbers and the real number system.

For example: rational and irrational numbers; properties (e.g., closure, distributive, associative) of the real number system and its subsets; operations and their inverses; the real number line; roots and powers; the laws of exponents; scientific notation; using number properties to prove theorems (e.g., the product of two even numbers is even); and problems involving real numbers and their operations.

PATTERNS, RELATIONS, AND ALGEBRA [12%]

0005 Understand and use patterns to model and solve problems.

For example: making conjectures about patterns presented in numeric, geometric, or tabular form; representing patterns and relations using symbolic notation; identifying patterns of change created by functions (e.g., linear, quadratic, exponential); and using finite and infinite series and sequences (e.g., Fibonacci, arithmetic, geometric) to model and solve problems.

0006 Understand how to manipulate and simplify algebraic expressions and translate problems into algebraic notation.

For example: the nature of a variable; evaluating algebraic expressions for a given value of a variable; the relationship between standard computational algorithms and algebraic processes; expressing direct and inverse relationships algebraically; expressing one variable in terms of another; manipulating and simplifying algebraic expressions; solving equations; and using algebraic expressions to model situations.

0007 Understand properties of functions and relations.

For example: the difference between functions and relations; the generation and interpretation of graphs that model real-world situations; multiple ways of representing functions (e.g., tabular, graphic, verbal, symbolic); properties of functions and relations (e.g., domain, range, continuity); piecewise-defined functions; addition, subtraction, and composition of functions; and graphs of functions and their transformations [e.g., the relationships among $f(x)$, $f(x + k)$, and $f(x) + k$].

0008 Understand properties and applications of linear relations and functions.

For example: the relationship between linear models and rate of change; direct variation; graphs of linear equations; slope and intercepts of lines; finding an equation for a line; methods of solving systems of linear equations and inequalities (e.g., graphing, substitution); and modeling and solving problems using linear functions and systems.

0009 Understand properties and applications of quadratic relations and functions.

For example: methods of solving quadratic equations and inequalities (e.g., factoring, completing the square, quadratic formula, graphing); real and complex roots of quadratic equations; graphs of quadratic functions; quadratic maximum and minimum problems; and modeling and solving problems using quadratic relations, functions, and systems.

0010 Understand properties and applications of exponential, polynomial, rational, and absolute value functions and relations.

For example: problems involving exponential growth (e.g., population growth, compound interest) and decay (e.g., half-life); inverse variation; modeling problems using rational functions; properties and graphs of polynomial, rational, and absolute value functions; and the use of graphing calculators and computers to find numerical solutions to problems involving exponential, polynomial, rational, and absolute value functions.

GEOMETRY AND MEASUREMENT [9%]

0011 Understand principles, concepts, and procedures related to measurement.

For example: using appropriate units of measurement; unit conversions within and among measurement systems; problems involving length, area, volume, mass, capacity, density, time, temperature, angles, and rates of change; problems involving similar plane figures and indirect measurement; the effect of changing linear dimensions on measures of length, area, or volume; and the effects of measurement error and rounding on computed quantities (e.g., area, density, speed).

0012 Understand the principles of Euclidean geometry and use them to prove theorems.

For example: the nature of axiomatic systems; undefined terms and postulates of Euclidean geometry; relationships among points, lines, angles, and planes; methods for proving triangles congruent; properties of similar triangles; justifying geometric constructions; proving theorems within the axiomatic structure of Euclidean geometry; and the origins and development of geometry in different cultures (e.g., Greek, Hindu, Chinese).

0013 Apply Euclidean geometry to analyze the properties of two-dimensional figures and to solve problems.

For example: using deduction to justify properties of and relationships among triangles, quadrilaterals, and other polygons (e.g., length of sides, angle measures); identifying plane figures given characteristics of sides, angles, and diagonals; the Pythagorean theorem; special right triangle relationships; arcs, angles, and segments associated with circles; deriving and applying formulas for the area of composite shapes; and modeling and solving problems involving two-dimensional figures.

0014 Solve problems involving three-dimensional shapes.

For example: area and volume of and relationships among three-dimensional figures (e.g., prisms, pyramids, cylinders, cones); perspective drawings; cross sections (including conic sections) and nets; deriving properties of three-dimensional figures from two-dimensional shapes; and modeling and solving problems involving three-dimensional geometry.

0015 Understand the principles and properties of coordinate and transformational geometry.

For example: representing geometric figures (e.g., triangles, circles) in the coordinate plane; using concepts of distance, midpoint, slope, and parallel and perpendicular lines to classify and analyze figures (e.g., parallelograms); characteristics of dilations, translations, rotations, reflections, and glide-reflections; types of symmetry; properties of tessellations; transformations in the coordinate plane; and using coordinate and transformational geometry to prove theorems and solve problems.

DATA ANALYSIS, STATISTICS, AND PROBABILITY [6%]

0016 Understand descriptive statistics and the methods used in collecting, organizing, reporting, and analyzing data.

For example: constructing and interpreting tables, charts, and graphs (e.g., line plots, stem-and-leaf plots, box plots, scatter plots); measures of central tendency (e.g., mean, median, mode) and dispersion (e.g., range, standard deviation); frequency distributions; percentile scores; the effects of data transformations on measures of central tendency and variability; evaluating real-world situations to determine appropriate sampling techniques and methods for gathering and organizing data; making appropriate inferences, interpolations, and extrapolations from a set of data; interpreting correlation; and problems involving linear regression models.

0017 Understand the fundamental principles of probability.

For example: representing possible outcomes for a probabilistic situation; counting strategies (e.g., permutations and combinations); computing theoretical probabilities for simple and compound events; using simulations to explore real-world situations; connections between geometry and probability (e.g., probability as a ratio of two areas); and using probability models to understand real-world phenomena.

TRIGONOMETRY, CALCULUS, AND DISCRETE MATHEMATICS [5%]

0018 Understand the properties of trigonometric functions and identities.

For example: degree and radian measure; right triangle trigonometry; the law of sines and the law of cosines; graphs and properties of trigonometric functions and their inverses; amplitude, period, and phase shift; trigonometric identities; and using trigonometric functions to model real-world periodic phenomena.

0019 Understand the conceptual basis of calculus.

For example: the concept of limit; the relationship between slope and rates of change; how the derivative relates to maxima, minima, points of inflection, and concavity of curves; the relationship between integration and the area under a curve; modeling and solving basic problems using differentiation and integration; and the development of calculus.

0020 Understand the principles of discrete/finite mathematics.

For example: properties of sets; recursive patterns and relations; problems involving iteration; properties of algorithms; finite differences; linear programming; properties of matrices; and characteristics and applications of graphs and trees.

HISTORY, PHILOSOPHY, AND METHODOLOGY OF SCIENCE [8%]

0021 Understand the nature of scientific thought and inquiry and the historical development of major scientific ideas.

For example: the reliance of scientific investigation on empirical data; the use of verifiable evidence, reasoning, and logical arguments; the importance of avoiding bias; the evaluation of scientific claims and arguments; science and technology in the ancient world (e.g., China, Greece); the foundations for modern science in the seventeenth and eighteenth centuries; the development of modern science in the nineteenth and twentieth centuries; key figures, discoveries, and theories (e.g., the Copernican revolution, Darwin's theory of evolution); and social, religious, and economic conditions that supported or inhibited the development of science and technology.

0022 Understand principles and procedures of research and experimental design.

For example: the formulation of testable hypotheses; the use of carefully planned research to solve problems; procedures and considerations, including validity and reliability, in setting up and conducting scientific investigations; the use of sampling techniques; and hypothesis testing using control and experimental groups.

0023 Understand procedures for gathering, organizing, interpreting, evaluating, and communicating scientific information.

For example: the systematic observation of phenomena; strategies, tools, and technologies for gathering, measuring, recording, and processing data; advantages and disadvantages of various measurement methods and devices; solving problems involving measurement; the use of various formats (e.g., graphs, flowcharts, tables, step-by-step directions, maps, reports) for organizing, communicating, and interpreting information; the use of data for making predictions and drawing conclusions; and developing models and statistical methods for interpreting and reporting data.

0024 Understand the safe and proper use of tools, equipment, and materials (including chemicals and living organisms) related to classroom and other science investigations.

For example: practices and requirements related to the safe use and storage of tools and equipment; the use and proper disposal of materials; procedures for preventing accidents in the science laboratory; procedures for dealing with accidents and injuries in the science laboratory; and proper practices and requirements related to the use and care of living organisms.

CHEMISTRY [8%]

0025 Understand the structure and nature of matter.

For example: the atomic and molecular structure of matter; the structure of the atom; the use of models of atomic structure to explain chemical behavior; the relationship between atomic structure and the organization of the periodic table; the difference between mixtures and pure substances; and chemical symbols, formulas, and equations.

0026 Understand the nature of physical changes in matter.

For example: states of matter and their characteristics; properties of common materials; physical properties and changes; changes of state and related changes in energy; the concept of mass; the principle of conservation of matter; and types and properties of mixtures and solutions.

0027 Understand the nature of chemical changes in matter.

For example: the description of chemical changes in terms of properties and composition of reactants and products; the use of the principle of conservation of matter to analyze chemical reactions; types of chemical bonds, their characteristics, and their effects on the characteristics of matter; and factors that affect rates of reaction.

0028 Understand the kinetic molecular model of matter.

For example: use of the kinetic molecular theory to explain the states of matter; interrelationships among pressure, temperature, and volume in gaseous systems; and the relationship between temperature and kinetic energy.

PHYSICS [8%]

0029 Understand the concepts of force, motion, work, and power.

For example: Newton's laws of motion; the relationship between mass and inertia; the difference between mass and weight; the vector nature of force, displacement, velocity, and acceleration; characteristics of force, work, and power; the motion of an object in terms of speed, velocity, acceleration, inertia, and momentum; distance-versus-time graphs; and the types and characteristics of simple machines.

0030 Understand the concept of energy and the forms that energy can take.

For example: the concept of conservation of energy; forms of energy (e.g., mechanical, light, thermal, electrical, nuclear); the classification of energy as kinetic or potential; the relationship between kinetic and potential energy; processes of energy transfer and conversion; elastic and inelastic collisions; and the concepts of entropy and thermodynamics.

0031 Understand characteristics of waves and the behavior of sound and light waves.

For example: transverse and longitudinal waves; characteristics (e.g., amplitude, wavelength, frequency) of waves and oscillations; the relationship of wave characteristics to wave speed and wave energy; the relationship between wave characteristics and properties of sound (e.g., loudness, pitch) and light (e.g., color, intensity); wave interactions; the properties and behavior of sound and light waves in various media; phenomena related to light and the behavior of light in various situations (e.g., refraction, diffraction, dispersion); and characteristics and properties of the electromagnetic spectrum.

0032 Understand principles of electricity, magnetism, and electromagnetism.

For example: the properties and formation of static electricity; characteristics of electron flow and electric current; characteristics and components (e.g., batteries, resistors) of simple electric circuits; the interpretation of electric circuit diagrams; characteristics of magnets and magnetic fields; and the principles of electromagnetism.

BIOLOGY [8%]

0033 Understand the characteristics and life processes of living organisms.

For example: differences between living organisms and nonliving things; basic cell structures and their functions; comparisons between animal cells and plant cells; growth of multicellular organisms by cell growth and reproduction; processes of photosynthesis and cellular respiration; homeostatic and metabolic processes; levels of biological organization (i.e., molecules, cells, tissues, organs, and systems); and structures and functions of major systems in plants and animals and interactions between the systems.

0034 Understand principles related to the inheritance of characteristics.

For example: how characteristics are passed on from generation to generation, including mutations and the influence of environmental factors on the inheritance of characteristics; and the structures and functions of DNA, genes, and chromosomes.

0035 Understand principles and theories related to biological evolution.

For example: theories and processes associated with the origin and evolution of life and scientific evidence for these theories and processes; methods used to investigate evolution; the roles of genetic and phenotypic variation, environmental factors, and natural selection in speciation; and the connection between evolutionary relationships and taxonomy.

0036 Understand characteristics of populations, communities, ecosystems, and biomes.

For example: biotic and abiotic factors that affect populations, communities, ecosystems, and biomes; strategies used by organisms to obtain basic requirements for life (e.g., food, shelter, oxygen, water); interrelationships among organisms, including humans, in ecosystems; energy transfers in food webs and food chains; the process of ecological succession; responses of ecosystems to change; and factors regulating population sizes within ecosystems.

EARTH AND SPACE SCIENCE [8%]

0037 Understand geologic history and processes related to the changing earth.

For example: theories of the origin and history of the earth; methods of determining the relative and absolute ages of inorganic and organic materials; the structure and composition of the earth and its layers; types and characteristics of minerals, rocks, and soils; the processes of mineral formation; processes that are involved in the formation and destruction of igneous, sedimentary, and metamorphic rock; the theory of plate tectonics and supporting evidence; processes of structural change of the earth's crust; the effects of various agents (e.g., glaciers, water, wind) on the earth's surface; important topographical features of the earth and their characteristics; types and characteristics of maps and map projections commonly used in science; and the effects of catastrophic phenomena (e.g., earthquakes, collisions with asteroids) on the earth and its inhabitants.

0038 Understand characteristics and properties of the hydrosphere.

For example: properties of water; characteristics of oceans, surface water, and ground water; and use of the water cycle to explain the movement and renewal of ground water and of water in oceans, glaciers, rivers, lakes, and watersheds.

0039 Understand the earth's atmosphere, weather, and climate.

For example: the structure and characteristics of the atmosphere; factors that contribute to the uneven heating of the earth's surface; the effects on weather of the uneven heating of the earth's surface; mechanisms of energy transfer in the atmosphere; air pressure and the movement of air in the atmosphere; cloud formation and precipitation; equipment and techniques used to monitor the weather; the interpretation of meteorological information; and techniques used to predict the weather and climatic change.

0040 Understand components of the solar system and universe and their interactions.

For example: the planets and their characteristics; interactions and movements of the earth, moon, and sun (e.g., seasons, moon phases, tides, eclipses); characteristics of stars and other objects in the solar system and universe; and theories of the origin and evolution of the universe.

INTEGRATION OF KNOWLEDGE AND UNDERSTANDING OF MATHEMATICS [10%]

In addition to answering multiple-choice items, candidates will prepare written responses to questions addressing content from the preceding mathematics objectives (1–20), which are summarized in the objective and descriptive statement below.

0041 Prepare an organized, developed analysis on a topic related to one or more of the following: number sense and operations; patterns, relations, and algebra; geometry and measurement; data analysis, statistics, and probability; and trigonometry, calculus, and discrete mathematics.

For example: presenting a detailed solution to a problem involving one or more of the following: place value, number base, and the structure and operations of number systems; application of ratios and proportions in a variety of situations; properties, attributes, and representations of linear functions; modeling problems using exponential functions; the derivative as a rate of change and the integral as area under the curve; applications of plane and three-dimensional geometry; and design, analysis, presentation, and interpretation of a statistical survey.

INTEGRATION OF KNOWLEDGE AND UNDERSTANDING OF SCIENCE [10%]

In addition to answering multiple-choice items, candidates will prepare written responses to questions addressing content from the preceding science objectives (21–40), which are summarized in the objective and descriptive statement below.

0042 Prepare an organized, developed analysis on a topic related to one or more of the following: history, philosophy, and methodology of science; chemistry; physics; biology; and earth and space science.

For example: the structure and nature of matter; chemical and physical changes in matter; the concepts of energy, force, and motion; characteristics and behavior of waves, sound, and light; characteristics and life processes of living organisms; principles and theories related to the inheritance of characteristics and biological evolution; the structure and composition of the earth and processes of structural change in the earth's crust; characteristics and properties of the hydrosphere and atmosphere; components of the solar system and universe and their interactions; and the nature of scientific thought and inquiry.

FORMULAS

Description	Formula
Sum of the measures of the interior angles in a polygon	$S = (n - 2) \times 180$
Circumference of a circle	$C = 2\pi r$
Area of a circle	$A = \pi r^2$
Area of a triangle	$A = \frac{1}{2}bh$
Surface area of a sphere	$A = 4\pi r^2$
Lateral surface area of a right circular cone	$A = \pi r\sqrt{r^2 + h^2}$
Surface area of a cylinder	$A = 2\pi rh + 2\pi r^2$
Volume of a sphere	$V = \frac{4}{3}\pi r^3$
Volume of a right cone and a pyramid	$V = \frac{1}{3}Bh$
Volume of a cylinder	$V = \pi r^2 h$
Sum of an arithmetic series	$S_n = \frac{n}{2}[2a + (n - 1)d] = n\left(\frac{a + a_n}{2}\right)$
Sum of a geometric series	$S_n = \frac{a(1 - r^n)}{1 - r}$
Sum of an infinite geometric series	$\sum_{n=0}^{\infty} ar^n = \frac{a}{1 - r}, \lvert r \rvert < 1$
Distance formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Midpoint formula	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

FORMULAS (continued)

Description	Formula
Slope	$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
Law of sines	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Law of cosines	$c^2 = a^2 + b^2 - 2ab \cos C$
Arc length	$s = r\theta$
Density of an object	$D = \frac{m}{V}$
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

PERIODIC TABLE OF THE ELEMENTS

1 **IA**																	18 **VIIIA**
1 **H** 1.01	2 **IIA**											13 **IIIA**	14 **IVA**	15 **VA**	16 **VIA**	17 **VIIA**	2 **He** 4.00
3 **Li** 6.94	4 **Be** 9.01											5 **B** 10.81	6 **C** 12.01	7 **N** 14.01	8 **O** 16.00	9 **F** 19.00	10 **Ne** 20.18
11 **Na** 23.0	12 **Mg** 24.3	3 **IIIB**	4 **IVB**	5 **VB**	6 **VIB**	7 **VIIB**	8	9 **VIIIB**	10	11 **IB**	12 **IIB**	13 **Al** 27.0	14 **Si** 28.1	15 **P** 31.0	16 **S** 32.1	17 **Cl** 35.5	18 **Ar** 39.9
19 **K** 39.1	20 **Ca** 40.1	21 **Sc** 45.0	22 **Ti** 47.9	23 **V** 50.9	24 **Cr** 52.0	25 **Mn** 54.9	26 **Fe** 55.8	27 **Co** 58.9	28 **Ni** 58.7	29 **Cu** 63.5	30 **Zn** 65.4	31 **Ga** 69.7	32 **Ge** 72.6	33 **As** 74.9	34 **Se** 79.0	35 **Br** 79.9	36 **Kr** 83.8
37 **Rb** 85.5	38 **Sr** 87.6	39 **Y** 88.9	40 **Zr** 91.2	41 **Nb** 92.9	42 **Mo** 95.9	43 **Tc** 98.9	44 **Ru** 101.1	45 **Rh** 102.9	46 **Pd** 106.4	47 **Ag** 107.9	48 **Cd** 112.4	49 **In** 114.8	50 **Sn** 118.7	51 **Sb** 121.8	52 **Te** 127.6	53 **I** 126.9	54 **Xe** 131.3
55 **Cs** 132.9	56 **Ba** 137.3	57–71	72 **Hf** 178.5	73 **Ta** 180.9	74 **W** 183.9	75 **Re** 186.2	76 **Os** 190.2	77 **Ir** 192.2	78 **Pt** 195.1	79 **Au** 197.0	80 **Hg** 200.6	81 **Tl** 204.4	82 **Pb** 207.2	83 **Bi** 209.0	84 **Po** (209)	85 **At** (210)	86 **Rn** (222)
87 **Fr** (223)	88 **Ra** (226)	89–103	104 **Rf** (261)	105 **Db** (262)	106 **Sg** (263)	107 **Bh** (264)	108 **Hs** (265)	109 **Mt** (268)	110	111	112	113	114	115	116	117	118

Lanthanide Series	57 **La** 138.9	58 **Ce** 140.1	59 **Pr** 140.9	60 **Nd** 144.2	61 **Pm** (145)	62 **Sm** 150.4	63 **Eu** 152.0	64 **Gd** 157.3	65 **Tb** 158.9	66 **Dy** 162.5	67 **Ho** 164.9	68 **Er** 167.3	69 **Tm** 168.9	70 **Yb** 173.0	71 **Lu** 175.0
Actinide Series	89 **Ac** (227)	90 **Th** 232.0	91 **Pa** 231.0	92 **U** 238.0	93 **Np** (237)	94 **Pu** (244)	95 **Am** (243)	96 **Cm** (247)	97 **Bk** (247)	98 **Cf** (251)	99 **Es** (252)	100 **Fm** (257)	101 **Md** (258)	102 **No** (259)	103 **Lr** (262)

Some of the elements 110 and above have been reported but not fully authenticated and named.

1. **Use the sequence of steps below to answer the question that follows.**

$$
\begin{aligned}
4.26 \times 2.2 &= \left(426 \times \tfrac{1}{100}\right) \times \left(22 \times \tfrac{1}{10}\right) \\
&= 426 \times \left(\tfrac{1}{100} \times 22\right) \times \tfrac{1}{10} \\
&= 426 \times \left(22 \times \tfrac{1}{100}\right) \times \tfrac{1}{10} \\
&= (426 \times 22) \times \left(\tfrac{1}{100} \times \tfrac{1}{10}\right) \\
&= (426 \times 22) \times \left(\tfrac{1}{1000}\right)
\end{aligned}
$$

The sequence of steps above could be used to answer which of the following questions?

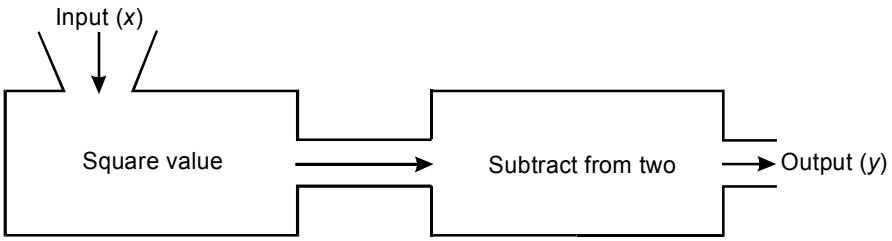
A. How can a decimal be converted to a fraction reduced to lowest terms?

B. Where should the decimal point be placed in the product of two decimals?

C. Why is it necessary to invert and multiply when solving problems involving fractions?

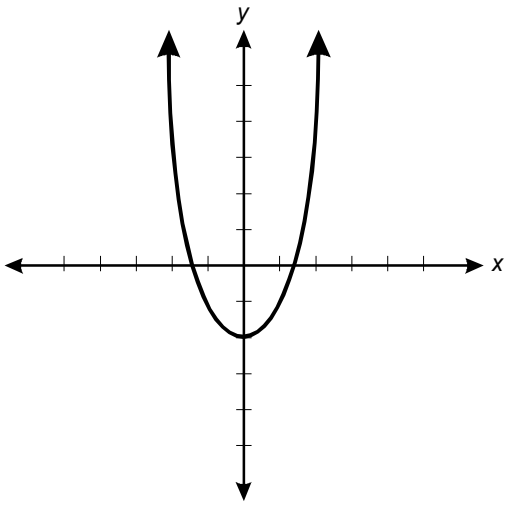
D. How is scientific notation used to multiply decimals?

2. **Use the diagram below to answer the question that follows.**

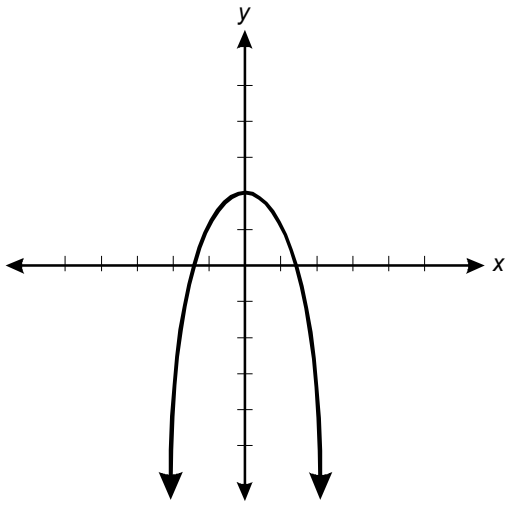


Which of the following graphs represents the function described in the above diagram?

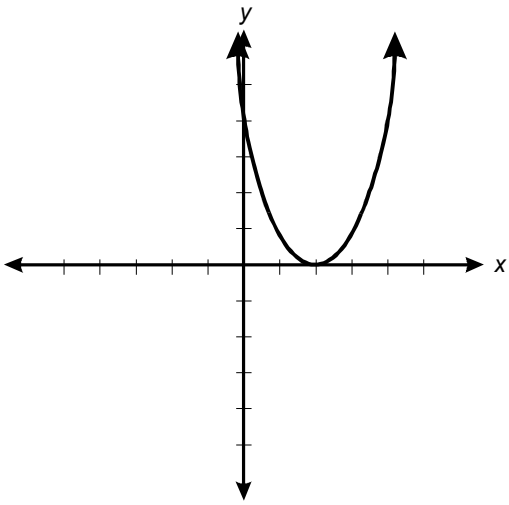
A.



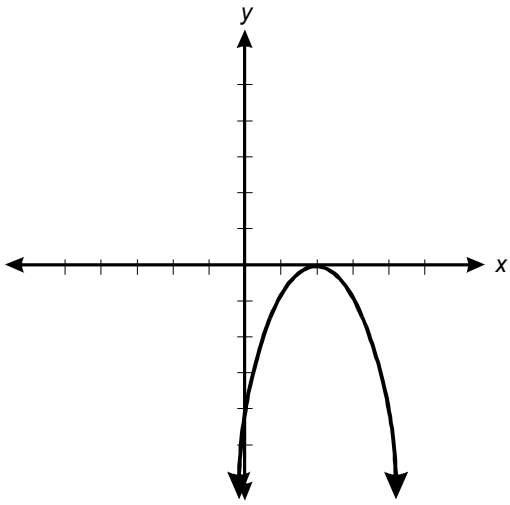
B.



C.



D.

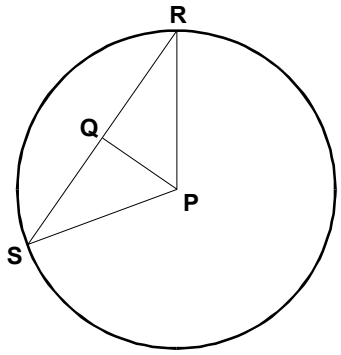


3. **Use the geometric proof below to answer the question that follows.**

Given: Circle P;

$\overline{PQ}$ is a median of $\triangle PRS$

Prove: $\triangle RPQ \cong \triangle SPQ$



Statements	**Reasons**
1. Circle P; $\overline{PQ}$ is a median of $\triangle PRS$	1. Given
2. $\overline{RQ} \cong \overline{QS}$	2. Definition of median
3. $\overline{PR}$ and $\overline{PS}$ are radii of P.	3. Definition of radius
4. $\overline{PR} \cong \overline{PS}$	4. All the radii of a circle are congruent.
5. ?	5. ?
6. $\triangle RPQ \cong \triangle SPQ$	6. SSS postulate

Which of the following statements and reasons would be most appropriate in step 5 of this proof?

A. **Statement** $\angle QPR \cong \angle QPS$ **Reason** $\overline{PQ} \perp \overline{RS}$

B. **Statement** P is the center of the circle. **Reason** Definition of a circle

C. **Statement** $\angle PSQ \cong \angle PRQ$ **Reason** Properties of isosceles triangle

D. **Statement** $\overline{PQ} \cong \overline{PQ}$ **Reason** Reflexive property of congruence

4. Which of the following situations best represents a random sampling?

   A. Ask every tenth person coming out of a health spa how many times a week they exercise to determine how often people in the town exercise.

   B. Survey students in advanced biology classes to determine the average amount of time students in a certain school study each week.

   C. Find the heights of all boys in a senior gym class to determine the average height of all boys in the school.

   D. Count the number of chocolate chips in every fifth cookie to determine the average number of chocolate chips per cookie in a bag of cookies.

5. The graph of the function $f(x) = 2x^2 - 6x + 4$ has its minimum value at the point $\left(\frac{3}{2}, -\frac{1}{2}\right)$. Which of the following statements must be true?

   A. The derivative of $f(x)$ is zero when x is equal to $\frac{3}{2}$.

   B. The value of $f(x)$ approaches a limit of zero as x approaches $\frac{3}{2}$.

   C. The graph of $f(x)$ shifts from a concave downward shape to a concave upward shape.

   D. The slope of the line tangent to $f(x)$ at $x = \frac{3}{2}$ is equal to $-\frac{1}{2}$.

6. Students are planning to study the effects of a nitrogen fertilizer on the growth of young corn plants. Two weeks earlier, the students started approximately 100 plants from seed under identical conditions. From this group of 100 seedlings, the students will select 40 seedlings to use in their study. To provide experimental results that will be most representative of corn seedlings in general, the students should choose:

   A. 20 large and 20 small seedlings.

   B. 40 mid-sized seedlings.

   C. 40 randomly selected seedlings.

   D. the 40 smallest-looking seedlings.

7. Containers 1 and 2 are identical. Container 1 contains one mole of gas X, and container 2 contains one mole of gas Y. Both containers are sealed and represent closed systems. The molecules of the gases in the two containers have the same average speeds, but gas Y has a much higher temperature than gas X. Which of the following pairs of gases is possible for gas X and gas Y?

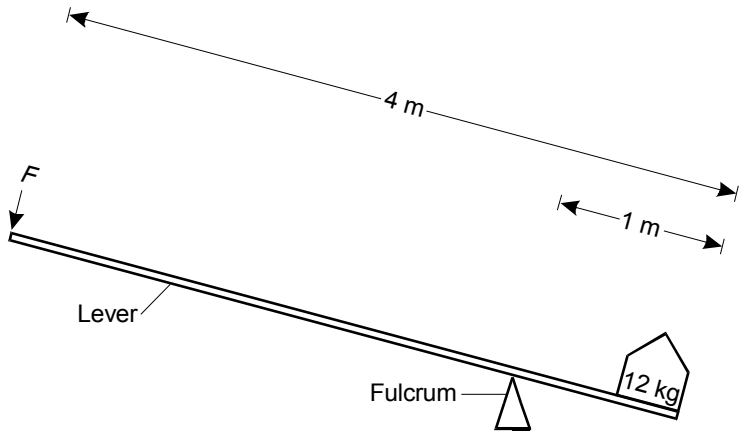
   A. Gas X is helium, and gas Y is hydrogen.

   B. Gas X is argon, and gas Y is hydrogen.

   C. Gas X is oxygen, and gas Y is helium.

   D. Gas X is oxygen, and gas Y is argon.

8. **Use the diagram below to answer the question that follows.**



The lever shown in the diagram above is 4 m long, with the fulcrum 1 m from one end. A 12 kg load is placed on the end of the lever nearest the fulcrum. What load must be applied to the opposite end of the lever at *F* in order to balance the 12 kg load?

A. 3 kg

B. 4 kg

C. 8 kg

D. 9 kg

9. A genetic mutation that results in a nucleotide insertion usually has a much greater effect on the organism than a genetic mutation that results in a nucleotide substitution. This is true because a nucleotide insertion:

   A. causes a shift in the reading frame of the DNA code.

   B. causes structural damage to the DNA molecule.

   C. makes a gene unreadable at the ribosomal sites.

   D. disrupts normal processes of mitosis and meiosis.

10. **Use the information below to answer the question that follows.**

    As Pangaea pulled apart, the modern Atlantic Ocean was formed. At the same time, another rift developed just to the west, along the present-day Connecticut River Valley. This rift, rather than forming an ocean or sea, resulted in volcanic activity and lava flows in the area that is known today as the Holyoke Range. Much of the old rift has since been filled in by sediment that later lithified.

    Based on the information in this passage, the rock making up the Holyoke Range can best be classified as:

    A. aggregate.

    B. sedimentary rock.

    C. metamorphic rock.

    D. igneous rock.

11. **Use the information below to complete the exercise that follows.**

A company is considering two bonus-plan options for its employees for the next 20 years. The two options are explained in the following chart.

Option 1:	Receive $2 the first year. Every year thereafter receive twice the bonus amount of the previous year.
Option 2:	Receive $200 the first year. Every year thereafter receive $200 more than the bonus amount of the previous year.

Use your knowledge of exponential and linear functions to develop a response in which you analyze the bonus received each year during a 20-year period under each option. In your response:

- create a data table representing the bonus received each year over a 12-year period for each option;
- graph the data from both tables on the same coordinate grid and connect the data with the line or curve that best fits the data;
- compare the bonus plans over the 12-year period, including a discussion of the significance of the point of intersection of the two graphs;
- explain what type of function, exponential or linear, models each option;
- find equations that describe each option; and
- identify an expression that represents the difference between the bonuses received under the two options in the twentieth year.

Be sure to show your work and explain the reasoning you use in analyzing and solving this problem.

12. **Use the information below to complete the exercise that follows.**

In the lungs of mammals, including humans, the circulatory and respiratory systems are closely associated anatomically and physiologically.

Using your knowledge of the circulatory and respiratory systems, write an essay in which you:

- describe what important physiological process occurs in the lungs;
- explain how the structures of the lungs and the circulatory system facilitate this process; and
- briefly describe how pulmonary circulation differs from systemic circulation.

Answer Key and Sample Responses: Middle School Mathematics/Science (51)

Question Number	Correct Response	Test Objective
1.	**B**	Understand principles and operations related to integers, fractions, decimals, percents, ratios, and proportions.
2.	**B**	Understand properties of functions and relations.
3.	**D**	Understand the principles of Euclidean geometry and use them to prove theorems.
4.	**D**	Understand descriptive statistics and the methods used in collecting, organizing, reporting, and analyzing data.
5.	**A**	Understand the conceptual basis of calculus.
6.	**C**	Understand principles and procedures of research and experimental design.
7.	**D**	Understand the kinetic molecular model of matter.
8.	**B**	Understand the concepts of force, motion, work, and power.
9.	**A**	Understand principles related to the inheritance of characteristics.
10.	**D**	Understand geologic history and processes related to the changing earth.

11. Sample Response

The sample response below reflects a strong knowledge and understanding of the subject matter.

The data tables representing the bonuses received each year over a 12-year period for the two options are shown below.

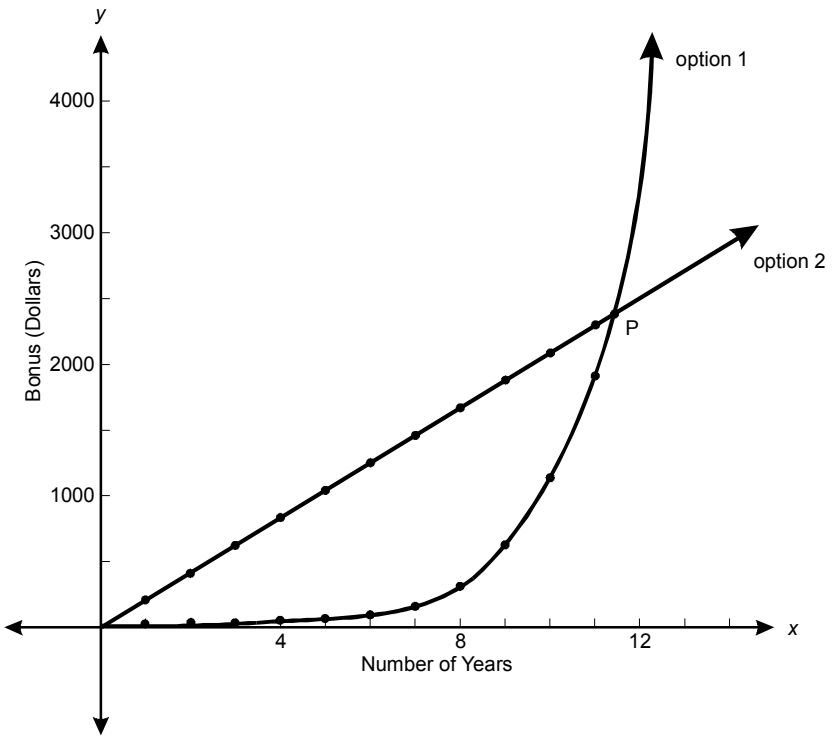
Option 1

Year	1	2	3	4	5	6	7	8	9	10	11	12
Bonus (dollars)	2	4	8	16	32	64	128	256	512	1024	2048	4096

Option 2

Year	1	2	3	4	5	6	7	8	9	10	11	12
Bonus (dollars)	200	400	600	800	1000	1200	1400	1600	1800	2000	2200	2400

The data from the tables can be graphed on a coordinate grid as shown below.



(continued)

The two graphs intersect at point P, where they both pay the same bonus. For all years before this point (approximately years 1 through 11), option 2 yields a higher bonus. For all years after this point (years 12 and beyond), option 1 yields a higher bonus.

The bonus plan offered in option 1 increases by a constant factor of 2 and therefore is modeled by an exponential equation. The exponential equation that models this bonus plan is $y = 2^x$. The bonus plan offered in option 2 increases by a constant rate of \$200 each year and therefore is modeled by a linear equation. The linear equation that models this bonus plan is $y = 200x$.

In the 20^{th} year, the bonus offered by option 1 exceeds that offered by option 2 by $2^{20} - 4000$ dollars.

12. Sample Response

The sample response below reflects a strong knowledge and understanding of the subject matter.

The lungs carry out the first stages of the physiological process of respiration, which carries oxygen to the cells of the body and removes carbon dioxide.

Inhaled air travels through the trachea, bronchi, bronchioles, and into the alveoli. The alveoli are air-filled sacs with walls that are only one cell thick and are surrounded by tiny capillaries filled with blood. Carbon dioxide dissolved in the blood diffuses across the walls of the alveoli into the air of the lungs and is exhaled. Oxygen diffuses from the air in the lungs across the alveolar walls and into the blood. This oxygen-rich blood travels to the heart as part of the pulmonary circulation and is then pumped to all the cells of the body as part of the systemic circulation.

The pulmonary circulation and the systemic circulation differ primarily in the relative amounts of oxygen and carbon dioxide in the arteries and veins. Blood that is low in oxygen and high in carbon dioxide is pumped through the pulmonary arteries to the capillaries surrounding the alveoli of the lungs. In the lungs, the blood loses carbon dioxide, picks up oxygen, and returns to the heart through the pulmonary veins. This blood, which is oxygen-rich and carbon dioxide-poor, is then pumped by the heart through the systemic arteries to the capillary beds of the body where it loses oxygen to the cells and picks up carbon-dioxide. The blood, which is now oxygen-poor and carbon dioxide-rich, then returns to the heart through the systemic veins and is pumped by the heart back to the lungs.