Massachusetts Tests for Educator Licensure® (MTEL®)

FIELD 63: MATHEMATICS
TEST OBJECTIVES

Subarea	Multiple-Choice	Range of Objectives	Approximate Test Weighting*
I.	Number Sense and Operations	01–02	10%
II.	Relations, Functions, and Algebra	03–08	30%
III.	Geometry and Measurement	09–12	20%
IV.	Probability, Statistics, Calculus, and Discrete Mathematics	13–17	20%
			80%
	Open-Response		
V.	Integration of Knowledge and Understanding	18–19	**20%**

*Final decisions regarding the proportion of the multiple-choice and open-response sections of the test will be made by the Department of Elementary and Secondary Education. If the proportions of the multiple-choice and open-response sections change, the proportions for the multiple-choice sections for each subarea will remain relative to the proportions indicated above.

DRAFT

Massachusetts Tests for Educator Licensure® (MTEL®)
Test Objectives
Field 63: Mathematics

SUBAREAS:

NUMBER SENSE AND OPERATIONS
RELATIONS, FUNCTIONS, AND ALGEBRA
GEOMETRY AND MEASUREMENT
PROBABILITY, STATISTICS, CALCULUS, AND DISCRETE MATHEMATICS
INTEGRATION OF KNOWLEDGE AND UNDERSTANDING

NUMBER SENSE AND OPERATIONS [10%]

0001 Apply knowledge of the properties and structure of the real number system.

For example:

- Apply concepts of place value, order, magnitude, absolute value, estimation (e.g., of irrational numbers, to judge reasonableness of solutions), and relationships between operations (e.g., division as the inverse of multiplication) to mathematical and real-world problems.
- Solve problems involving integers, fractions, decimals, percents, integer exponents, and rational exponents.
- Apply knowledge of multiple representations of numbers and number operations (e.g., scientific notation, area models for multiplication, laws of exponents, roots) to mathematical and real-world problems.
- Analyze and apply knowledge of the structure and properties (e.g., closure, distributive, associative) of the real number system and its subsets (e.g., whole numbers, integers, rational numbers, irrational numbers).
- Apply the fundamental theorem of arithmetic and the Euclidean algorithm to problems involving prime and composite numbers, greatest common factors, least common multiples, square roots, and properties of divisibility.

0002 Apply knowledge of the properties and structure of the complex number system and linear algebra.

For example:

- Represent complex numbers and their operations using multiple representations (e.g., vector, trigonometric, polar).
- Perform arithmetic operations on complex numbers and their conjugates, including calculating their roots and powers.
- Apply knowledge of the fundamental theorem of algebra to solve quadratic equations with real coefficients that have complex solutions.
- Perform operations on matrices and apply basic properties (e.g., associativity, commutativity, invertibility) of matrices and determinants to model and solve problems (e.g., solve a system of two linear equations, represent a linear transformation).
- Apply geometric and algebraic properties of vectors and vector arithmetic to model and solve problems.

RELATIONS, FUNCTIONS, AND ALGEBRA [30%]

0003 Analyze and apply algebraic techniques to expressions, equations, and inequalities.

For example:

- Apply properties of real numbers in algebraic contexts to manipulate and simplify algebraic expressions and solve equations and inequalities (e.g., linear, absolute value, rational, radical).
- Translate between situations presented as verbal descriptions and as algebraic sentences.
- Construct algebraic expressions or equations that model mathematical situations (e.g., symbolically, numerically, graphically).
- Analyze and justify algebraic techniques using properties of the real number system.
- Apply and analyze methods of solving quadratic equations and inequalities (e.g., factoring, completing the square, quadratic formula, graphing) to problems involving real and complex roots.
- Analyze and solve polynomial equations with real coefficients using algebraic methods (e.g., fundamental theorem of algebra, rational root theorem, conjugate root theorem).

0004 Apply the principles and properties of relations and functions.

For example:

- Identify, complete, and extend patterns presented numerically, geometrically, or in tabular form; represent them using symbolic notation; and analyze their properties and algebraic representations.
- Apply function notation and represent functions in various forms (e.g., tabular, graphic, symbolic, verbal, piecewise-defined).
- Analyze the properties of functions (e.g., domain and range, continuity, inverses, composition, transformations, difference between relations and functions) and apply arithmetic operations on functions.
- Analyze arithmetic and geometric sequences recursively and with an explicit formula, translate between the two forms, and use them to model and solve problems.

0005 Apply the principles and properties of linear, absolute value, and quadratic relations and functions.

For example:

- Represent linear, absolute value, and quadratic functions symbolically, numerically, graphically, and verbally.
- Analyze connections between proportional relationships, direct variation, rates of change, and linear models and use these connections to build linear functions.
- Apply properties of symmetry, roots, intercepts, and vertices to construct, analyze, and interpret quadratic relations and their graphs.
- Represent, solve, and analyze linear and quadratic systems of equations and inequalities algebraically and graphically, including those with infinitely many or no solutions.
- Model and solve problems involving absolute value functions and linear and quadratic relations, functions, and systems, including situations involving constraints.

0006 Apply the principles and properties of exponential and logarithmic relations and functions.

For example:

- Represent exponential and logarithmic relations and functions symbolically, numerically, graphically, and verbally.
- Analyze the relationship between exponential and logarithmic functions.
- Recognize and apply concepts of symmetry, roots, intercepts, and asymptotes to construct, analyze, and interpret exponential and logarithmic functions and inequalities and their graphs.
- Apply properties of exponents and logarithms to manipulate exponential and logarithmic expressions.
- Solve exponential and logarithmic equations and inequalities graphically and algebraically.
- Model and solve problems involving exponential and logarithmic functions and use them to make predictions in real-world and mathematical situations (e.g., compound interest, exponential growth and decay, amortization, annuities).

0007 Apply the principles and properties of polynomial, radical, and rational relations and functions.

For example:

- Represent polynomial, radical, and rational relations and functions symbolically, numerically, graphically, and verbally.
- Recognize and apply concepts of symmetry, roots, intercepts, critical points, and asymptotes to construct, analyze, and interpret polynomial, radical, and rational functions and inequalities and their graphs.
- Solve and analyze polynomial, radical, and rational equations and inequalities graphically, algebraically, and numerically.
- Apply algebraic properties of polynomial, radical, and rational functions (e.g., factoring, partial fractions, finite differences) to mathematical problems.
- Apply knowledge of polynomial, radical, and rational relations, functions, and systems to model and solve problems.
- Analyze connections between joint variation, inverse variation, and rational models and use these connections to build rational functions.

0008 Apply the principles and properties of trigonometric functions and identities.

For example:

- Represent trigonometric functions symbolically, numerically, graphically, and verbally.
- Analyze connections between trigonometric functions and the unit circle.
- Apply properties (e.g., amplitude, frequency, period, phase shift) of trigonometric functions and their inverses to construct, analyze, and interpret trigonometric functions, inverses, and graphs.
- Model periodic phenomena with trigonometric functions.
- Manipulate trigonometric expressions and solve equations.
- Verify trigonometric identities using algebraic techniques.

GEOMETRY AND MEASUREMENT [20%]

0009 Apply the principles, concepts, and procedures related to units and measurement.

For example:

- Convert measurements within various systems of measurement (e.g., imperial, metric) and use dimensional analysis and nonlinear measuring scales (e.g., Richter, decibel, pH) to solve real-world problems.
- Solve problems involving mass, capacity, density, time, temperature, degree and radian measures of angles, and rates of change.
- Explain how changing linear dimensions of a multidimensional object may affect length, area, and volume.
- Apply measurement formulas (e.g., length, area, volume) to regular, non-regular, and composite shapes, regions, and solids.
- Solve problems involving indirect measurement (e.g., similar polygons, trigonometric ratios, proportional reasoning).
- Analyze the effects of measurement error and rounding on computed quantities (e.g., area, density, speed).

0010 Apply the axiomatic structure of Euclidean geometry.

For example:

- Demonstrate understanding of relationships between points, lines, planes, rays, and angles.
- Apply knowledge of the axiomatic structure of Euclidean geometry (e.g., undefined terms, definitions, postulates, theorems) to develop and analyze direct and indirect methods of proof.
- Apply and justify properties of triangles (e.g., congruence conditions, interior angle sum theorem, triangle inequality, trigonometric ratios, law of sines, law of cosines, Pythagorean theorem and its converse, special triangles).
- Apply and justify properties of polygons (e.g., sum of interior angles, properties of quadrilaterals).
- Apply and justify properties of circles (e.g., tangents, chords, arcs, inscribed angles).
- Demonstrate understanding of the derivation of area formulas.
- Identify and justify straightedge and compass constructions.

0011 Apply the principles and properties of Euclidean geometry to solve problems involving two- and three-dimensional objects.

For example:

- Characterize geometric relationships between lines and planes in three dimensions (e.g., parallel, perpendicular, skew, coplanar).
- Translate between two- and three-dimensional representations of geometric figures (e.g., cross sections, nets, projections, perspective drawings) and generate three-dimensional figures from two-dimensional shapes.
- Apply properties of two- and three-dimensional figures to solve real-world and mathematical problems.
- Apply concepts of similarity and congruence to analyze the properties and compare the measures (e.g., perimeter, area, volume) of two- and three-dimensional figures.

0012 Apply the principles and properties of coordinate and transformational geometry and the characteristics of non-Euclidean geometries.

For example:

- Classify, represent, and analyze geometric figures (e.g., lines, triangles, quadrilaterals, circles) in the coordinate plane, including translating between geometric descriptions and equations for conic sections.
- Analyze figures in two and three dimensions using coordinate methods (e.g., distance, midpoint, slope, parallel and perpendicular lines).
- Model and solve real-world and mathematical problems involving conic sections using coordinate methods.
- Apply knowledge of isometries and dilations in two- and three-dimensional space (e.g., symmetry, rotations, translations, reflections, scale factors), including their basic properties in relation to congruence, similarity, area, and volume.
- Demonstrate knowledge of axioms and key features (e.g., parallelism, angle sums, great circles, congruence) of spherical and hyperbolic geometries.

PROBABILITY, STATISTICS, CALCULUS, AND DISCRETE MATHEMATICS [20%]

0013 Apply the principles, properties, and techniques of probability.

For example:

- Analyze and determine simulations to model simple theoretical and experimental probabilities.
- Analyze situations and determine probabilities using a variety of representations (e.g., tables, area models, Venn diagrams, tree diagrams, sample spaces, normal models).
- Analyze, calculate, and interpret probabilities for simple and compound events using addition and multiplication rules.
- Apply knowledge of expected value to analyze problems and situations (e.g., fairness of games, lotteries), determine the probability of events, and evaluate outcomes of decisions.

0014 Apply the principles and concepts of descriptive statistics to the problem-solving process.

For example:

- Evaluate sampling techniques (e.g., random, systematic, cluster, stratified) and statistical methods and procedures (e.g., blocking, randomization, control, blinding) used for collecting data.
- Analyze the distribution of univariate data in terms of shape (e.g., uniformity, symmetry, skewness, modality), measures of central tendency (e.g., mean, median), and variation (e.g., interquartile range, variance, standard deviation), and describe effects of data transformations on measures of central tendency and variability.
- Select, justify, and interpret appropriate representations of data using a variety of displays (e.g., bar graphs, scatter plots, box plots, stem-and-leaf plots, histograms, two-way frequency tables).
- Interpret scatter plots and linear models to investigate patterns of association between two variables (e.g., correlation coefficient, coefficient of determination, impact of outliers on association).
- Evaluate the construction of confidence intervals (e.g., margins of error, confidence levels) and hypothesis tests (e.g., null and alternative hypotheses, *P*-values) and interpret their results.

0015 Apply principles and techniques of limits, continuity, and differential calculus.

For example:

- Find the limit of algebraic expressions, sequences, and graphic representations.
- Apply the concept of a limit to analyze properties of functions (e.g., continuity, asymptotes) and series.
- Apply the concept of a derivative (e.g., slope of a tangent line, limit of difference quotients, average and instantaneous rates of change) graphically, numerically, and analytically.
- Apply techniques of differentiation (e.g., product rule, quotient rule, chain rule) to find the derivative of a function.
- Apply principles of differential calculus to model and solve problems (e.g., velocity, acceleration, optimization, related rates) and analyze functions and planar curves (e.g., maxima/minima, concavity, tangent line approximations).

0016 Apply principles and techniques of integral calculus.

For example:

- Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., area approximations, limit of Riemann sums).
- Apply techniques of integration (e.g., *u*-substitution, integration by parts) to find the antiderivative of a function and interpret its meaning.
- Apply the fundamental theorem of calculus to model and solve problems.
- Apply principles of integral calculus to solve problems (e.g., area, work, volume, arc length, displacement, velocity).

0017 Apply the properties and techniques of discrete mathematics.

For example:

- Use the principle of mathematical induction to prove statements about integers.
- Apply a variety of models (e.g., vertex-edge graphs, tree diagrams, arrays) to solve problems.
- Demonstrate knowledge of basic set theory (e.g., unions, intersections) and logic (e.g., if/then, if and only if).
- Apply the basic principles of counting techniques (e.g., combinations, permutations) to represent and solve problems.

INTEGRATION OF KNOWLEDGE AND UNDERSTANDING [20%]

In addition to answering multiple-choice items, candidates will prepare written responses to assignments addressing content summarized in the objectives below.

0018 Prepare an organized, developed analysis on a topic integrating knowledge from at least two of the following: number sense and operations; relations, functions, and algebra; geometry and measurement; probability, statistics, calculus, and discrete mathematics.

For example:

- Write a formula, an equation, or equations to model a given situation.
- Create appropriate graphs and/or diagrams, including all proper labels, to model and describe the situation.
- Apply appropriate mathematical techniques to analyze the situation.
- Explain and justify the effects of changes to parameters in the situation.

0019 Prepare an organized, developed analysis on a topic integrating knowledge from at least two of the following: number sense and operations; relations, functions, and algebra; geometry and measurement; probability, statistics, calculus, and discrete mathematics.

For example:

- Create appropriate graphs and/or diagrams, including all proper labels, to model and describe a given real-world situation.
- Apply appropriate mathematical techniques to make a prediction or comparison regarding the situation.
- Make a recommendation or argument based on the prediction or comparison.
- Discuss factors that could influence the accuracy of the prediction/comparison and recommendation/argument.